

DYNAMIC RESPONSE OF AUTOMATICALLY  
CONTROLLED MOLE-DRAIN FLOW

ABSTRACT OF DISSERTATION

Presented in Partial Fulfillment of the Requirements for  
the Degree Doctor of Philosophy in the Graduate  
School of The Ohio State University

By

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The Ohio State University  
1971

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The purpose of this study was to develop a method for evaluating controllability of a given draintube plow and to determine the optimum mode of automatic grade control with a laserplane elevation reference system. The mathematical model developed for a floating-beam type mole-drain plow described the dynamics of moling depth as a function of the plow hitch motion. Regulation of the hitch height was assumed as the only means of controlling moling depth. A model was also developed for the laserplane automatic grade control system, in which the regulation of the hitch height (feedback) was expressed as a function of the displacement of the on-board laser receiver from the laserplane elevation datum. These mathematical models were nonlinear and did not lend themselves to analytical solutions. The plow and feedback control system was simulated on an analog computer for study.

A prototype mole plow was field tested to obtain basic steady-state and transient response parameters for use in the simulation. As expected, the draft force for the plow could be approximated with a power function of moling depth, and the level of action for the resultant draft force on the blade was at a constant fraction  $r$  of depth. Further, it was found that for a given plow configuration, the draft force acted at nearly a constant distance below the hitch for a significant range of moling depth.

The damping coefficient for the plow model was adjusted by trial-and-error such that the computer closely simulated the mole plow transient response determined in field tests. The damping coefficient was about 5.5 times the critical damping coefficient estimated by linearizing the plow model to a second-order equation. The simulation showed that typical hitch accelerations caused by the hydraulic depth adjusting mechanism had an insignificant effect on the moling depth. The plow model was, therefore, simplified by deleting the hitch acceleration term and modeling the hitch velocity as a pure on or off step-function.

Since the modeled mole plow operated in a slightly nonlinear manner, it was not possible to form a mole-drain channel with a given gradient by controlling the hitch point such that it followed a line parallel to the desired drain slope. A unit change in hitch height resulted in a  $(1/r)$  change in moling depth, where  $r$  is the fraction of depth at which the resultant draft force acts.

A simple on-off feedback control mode was found to be satisfactory, when the system gain was high enough to cause at least a one cycle per second limit-cycle motion of the hitch point. Dead-zone on-off, digital on-off, and proportional modes of control were also simulated for

comparison of results. The simulation showed that the mole channel could be held closest to the desired grade-line, independent of ground surface irregularities, when the laser receiver unit was mounted forward of the plow blade a distance equal to about one-sixth of the plow beam length. Only ramp, saw-tooth, and sine-wave functions, or their combination, were used as the simulated ground surface to evaluate the grade control system performance.

The simulation results indicated that the use of two laser receivers on the plow beam, one over the hitch and the other near the blade, would improve grade control accuracy. The proposed front receiver would provide the usual feedback control signal to the system hydraulics for regulation of the hitch height. The rear receiver would detect prolonged errors in moling depth and provide periodic step-wise signals to mechanically adjust (bias) the height of the front receiver above the hitch. This control system concept was recommended for further study.

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LIST OF SYMBOLS

<u>Symbol</u>	<u>Definition or Description</u>	<u>Units</u>
a	horizontal distance between the hitch and center-of-mass for the mole plow	ft.
b	effective beam length for the mole plow	ft.
B	plow beam-blade connection	-
C	damping coefficient for the mole plow model	$\frac{\text{lb.}}{\text{ft./sec.}}$
$C_c$	critical damping coefficient	$\frac{\text{lb.}}{\text{ft./sec.}}$
d	moling depth, or depth of soil cutting blade	ft.
$\dot{d}$	velocity of change in moling depth	ft./sec.
$\ddot{d}$	acceleration of change in moling depth	ft./sec. <sup>2</sup>
$d_0, d_1, d_2$	specific steady-state moling depths	ft.
$d_s, d_s'$	any given steady-state moling depth	ft.
$d_i, d_f$	initial and final moling depths, respectively	ft.
$d_p$	perturbation variation in moling depth	ft.
$d_{\text{crit.}}$	critical depth below which draft is linear	ft.
$d_{10\%}$	moling depth corresponding to the "10% settling distance" in mole plow response following a hitch displacement	ft.
$\Delta d$	increment of change in moling depth	ft.
f	a function of	-
$f_V(d)$	steady-state component of vertical soil resistance force which is a function of moling depth d	lb.

Symbol	Definition or Description	Units
$g$	gravitational acceleration	ft./sec. <sup>2</sup>
$G$	the center-of-gravity of the mole plow structure	-
$h$	hitch height	ft.
$\dot{h}$	velocity of vertical hitch motion	ft./sec.
$\ddot{h}$	acceleration of vertical hitch motion	ft./sec. <sup>2</sup>
$h_0, h_1, h_2$	specific steady-state hitch heights	ft.
$h_s, h_s'$	any given steady-state hitch height	ft.
$h_i, h_f$	initial and final hitch heights, respectively	ft.
$\Delta h$	increment of change in hitch height	ft.
$h_g$	simulated ground surface irregularities measured from the mean ground surface reference line	ft.
$\dot{h}_c$	velocity of hydraulic cylinder which controls the hitch height	ft./sec.
$H$	mole plow hitch point	-
$HBM$	mole plow hitch-beam-mole assembly	-
$J_G$	mass moment-of-inertia of the mole plow about its center-of-mass	lb.-sec. <sup>2</sup>
$J_H$	mass moment-of-inertia of the mole plow about the hitch point, equals $J_G + ma^2$	lb.-sec. <sup>2</sup>
$K$	coefficient of soil resistance	lb./ft.
$K_c$	gain factor for proportional control mode	$\frac{\text{ft./sec.}}{\text{ft.}}$
$K_s$	rate of change of soil cutting pressure with depth	lb./ft. <sup>3</sup>
$m$	mass of the mole plow structure	$\frac{\text{lb.}}{\text{ft./sec.}^2}$
$M$	base of the mole torpedo	-

Symbol	Definition or Description	Units
n	distance below the plow hitch H where the soil resistance force acts on the blade	ft.
N	normal soil force acting on the sides of the soil cutting wedge on the front of the blade	lb.
$N_1$	normal soil force on the side of the blade shank	lb.
p	exponent coefficient in draft equation	-
$p_s$	soil pressure on the cutting edge of the blade	lb./ft. <sup>2</sup>
P	pulling force at the plow hitch (drawbar pull)	lb.
$P_H$	horizontal component of pulling force (draft)	lb.
$P_V$	vertical component of pulling force	lb.
$P_{VS}$	steady-state vertical force at hitch	lb.
r	the fraction of the moling depth where the soil resistance force acts on the plow blade	-
$\left(-\frac{1}{r}\right)$	"effective" steady-state gain factor for the mole plow	-
R	soil resistance force	lb.
$R_H$	horizontal component of soil resistance force (draft)	lb.
$R_V$	vertical component of soil resistance force	lb.
s	gradient	ft./ft.
t	time	sec.
F	sliding friction force between soil and cutting edge of plow blade	lb.
$F_1$	sliding friction force along the sides of the plow blade shank	lb.
v	forward ground speed of mole plow and tractor	ft./sec.
(vT)	travel distance at speed v and for time interval T to evaluate error in moling depth	ft.

Symbol	Definition or Description	Units
w	thickness of plow blade	ft.
x	distance rearward from the hitch to the location of the laser receiver unit on the plow beam	ft.
y	vertical distance between the mole M and the blade-beam connection B	ft.
$y_o$	$y_o \triangleq h_o + d_o$ , where $h_o$ and $d_o$ are corresponding steady-state values and BH is horizontal	ft.
$z_o, z_s$	normal or steady-state height of the laser receiver above the mean ground surface reference plane	ft.
$z_p$	perturbation variation in the height of the receiver unit measured from $z_s$ or $z_o$	ft.
$z_{DZ}$	dead-zone movement of the receiver unit either above or below the "true null" for a dead-zone control mode	ft.
$z_{pl}$	proportional limit movement of the receiver unit either above or below the "true null" for a proportional controller with a limiter	ft.
$\alpha$	angle of soil cutting wedge on plow blade	deg.
$\beta$	plow beam-to-blade angle	deg.
$\epsilon$	error in moling depth	ft.
$\zeta$	damping ratio	-
$\theta$	angular rotation of the mole plow frame	rad.
$\dot{\theta}$	velocity of angular rotation of the plow	rad./sec.
$\ddot{\theta}$	acceleration of angular rotation of the plow	rad./sec. <sup>2</sup>
$\mu$	coefficient of soil-metal sliding friction	-
$\tau_L$	time lag in hydraulic cylinder-valve system	sec.
$\tau_s$	time delay before starting a correction in a digital on-off control mode	sec.
$\tau_r$	time duration for each rotation of the laser beam in the laserplane system	sec.

Symbol	Definition or Description	Units
$\phi$	angle of line-of-draft from horizontal	deg.
$\omega_n$	natural frequency of oscillation	rad./sec.
$\underline{\Delta}$	is defined as	-



## INTRODUCTION

The development of subsurface drainage materials and equipment to install agricultural drainage systems with rapidity and at low cost has long presented a challenge to engineers and inventors. Clay and concrete draintile have been the principal drainage materials for many decades, but material handling problems to install the conventional one-foot tile sections have been a major drawback to speeding up the installation operation. Perhaps this is one reason why the basic power trenching machine developed about the 1850 - 1875 era has changed very little.

About twenty years of research and development preceded the adoption and widespread use of plastic drainage tubing in the United States. Much of this research involved parallel developments of both drainage materials and experimental installation equipment. Furthermore, a majority of all investigative work on drainage equipment involved some modification of "mole drainage" because of its inherent high speed and the elimination of the usual slow ditching and backfilling operations common in drainage installations. The idea of "plowing-in" drain pipes was by no means new, even to investigators in the early 1900's. The method had been reportedly tried many times to install conventional clay draintile. One of the earliest such trials using a plow-type drainage machine was reported by French (1859) in his text Farm Drainage (pp. 247-248), where he gave an account of the "Fowler

Draining Plow", which was developed and tested in England. French described the plow's operation and the "claims" for it as follows:

"The pipes, of common drain tiles, are strung on a rope and this rope, with the pipes, is drawn through the ground, following a plug like the foot of a subsoil plow, leaving the pipes perfectly laid, and the drain completed at a single operation.." Fowler's plow was pulled by a horse powered cable windlass and a means of manually adjusting the plow blade was provided for controlling grade on the drain line being pulled in.<sup>1/</sup> As improved power sources became available, the combination 'mole-tile' system was tried many more times; one of the latest, and possibly most successful, being the German Poppelsdorf system reported by Wallen (1931). However, none of the methods developed ever met with widespread acceptance or use. This was probably because the approach was to utilize existing drainage materials, namely, ceramic tile, rather than some conduit material which would have been easier to handle. The introduction of light-weight, flexible plastic tubing into the United States during the early 1950's resulted in new investigations to be undertaken.

Schwab (1951, 1955) used a mole plow to pull experimental polyethylene plastic drain tubes into a mole-channel during experiments in Iowa from 1947 to 1954. Many other investigators followed, both in the U.S. and in Europe, each trying a different kind of plastic drain tube and/or installation method. In contrast to the use of preformed

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<sup>1/</sup> A complete accounting of the "Fowler Draining Plow" can also be found in: Weaver, M. M. "History of Tile Drainage," 1964, p. 178, fig. 102.

plastic tubing as tested by Schwab, a number of researchers developed plastic mole-drain "liners", which were formed from 'sheet' plastic material as the liner was inserted into the mole-channel behind a mole plow; e.g., Busch (1958), Fouss and Donnan (1962), Ede (1963), and Boa (1963). These plastic-lined mole drains did not prove to be strong enough to withstand soil loads over even a short 4 or 5 year test period, as later reported by Fouss (1965) and Ede (1965). The problems of controlling the high speed mole plow to install drains at specified depth and grade was recognized by each of these investigators. Thus, each conducted concurrent research to develop a method of grade-control for the plow. One of the first prototypes of a laser beam automatic grade-control system resulted from this work [Fouss and Fausey (1967)].

By mid-1967, corrugated-wall plastic drainage tubing had emerged as the most promising drainage conduit material for further development by the agricultural drainage industry [Fouss (1968)]. During the period 1967 - 1970, the fabrication and agricultural use of corrugated plastic drainage tubing increased very rapidly in the U.S.; in 1969, a fast growing market for the new drainage tubing began in Canada as well. In some areas of the midwestern U.S. during 1970, as much as one-third or more of the subsurface drainage systems installed on farms were with corrugated plastic drain tubing. Almost all of these plastic drain installations were made with modified trenching machines, with only a few "trial" installations or field demonstrations being made with "plow-type" drainage equipment. Several of the trenching machines installing the new tubing were equipped with the recently developed

"Laserplane"<sup>1/</sup> automatic grade control system, thus permitting faster digging speeds. Full advantage of the ease in material handling for the new corrugated plastic drainage tubing can be realized if the rapid "plow-in" method of installation is used. The draintube plow equipment, intended for larger scale drainage projects, will never receive widespread acceptance and use, however, if drains cannot be installed accurately to specified depth and grade. Much progress has been made, mostly by trial-and-error, to utilize the "Laserplane" auto-control system on floating-beam type plows. Many questions, however, remain unanswered on optimal mounting and operation of the auto-control system on drainage plows. The purpose of this study is to develop a method for evaluating controllability of a given draintube plow and to determine the optimum mode of auto-control.

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<sup>1/</sup> Laserplane Corp., Dayton, Ohio

## I. LITERATURE REVIEW

Investigation of subsoilers, mole plows, and various other drainage plows has been accomplished primarily by field or scale model tests. Considerable work has been done on shallow soil cutting and tillage tools, but the results are not directly applicable to drainage plows. Some of the pertinent concepts available in the literature are summarized below.

### A. Types of Mole Plows

Several types of mole plows have been developed but, basically, they fall into two classes as to the method of mole-depth control: Depth-gage wheel and floating-beam. An example depth-gage wheel plow, developed by the Caterpillar Tractor Co. (1958), is shown in Figure 1. This type is best suited where the land slope is uniform and constant depth operation is desired. On irregular ground surfaces, it would be extremely difficult to control the depth wheels with sufficient speed and accuracy to maintain grade in the drainage channel, particularly at normal ground speeds from 70 to 150 ft./min.

A floating-beam mole plow is better suited for operation on irregular ground surfaces. Mole plows of this type were developed and used in New Zealand about 1930. A tool-bar mounted, floating-beam mole plow developed by the Agricultural Research Service, USDA [Fouss (1968)], and adapted to install corrugated plastic drainage tubing, is shown in



6. Fig. 1. Depth-gage wheel controlled mole plow installing plastic-lined mole drain.

Figure 2. This plow is hitched at the sides of the crawler tracks to improve traction efficiency and dynamic stability of the tractor. Its operating depth is controlled by raising or lowering the plow hitch point with hydraulic cylinders. The hitch is situated about 12 ft. forward of the plow blade. The counteracting rotational moments about the hitch pin due to the plow weight and soil resistance (draft) balance each other and the plow is said to operate with a "floating-beam" action. Changes in the vertical position of the hitch are not immediately reflected in the operating depth of the blade and mole. The mole adjusts or "floats" to a new equilibrium depth as the tractor moves forward.

Another type of floating-beam mole plow, unique in design, was developed by Ede (1961) in England. A commercial version of this plow called the Badger Minor, is shown in Figure 3. The mole plow blade and the tractor are connected by a pair of rollers which run in a curved track that is mounted on the rear of the tractor. The center of curvature of this roller track acts as a virtual hitch point which coincides approximately with the center of the crawler tracks. The blade is thus nearly isolated from any pitching movements of the tractor. Depth and gradient regulation is achieved by raising and lowering the imaginary hitch point; this is done by hydraulically moving the roller track frame.

#### B. Grade Control of Drainage Equipment

Many attempts have been made to eliminate the need for setting grade-line targets and/or to automate the grading operation. Several systems adaptable to trenchers and plows have used a cord or wire

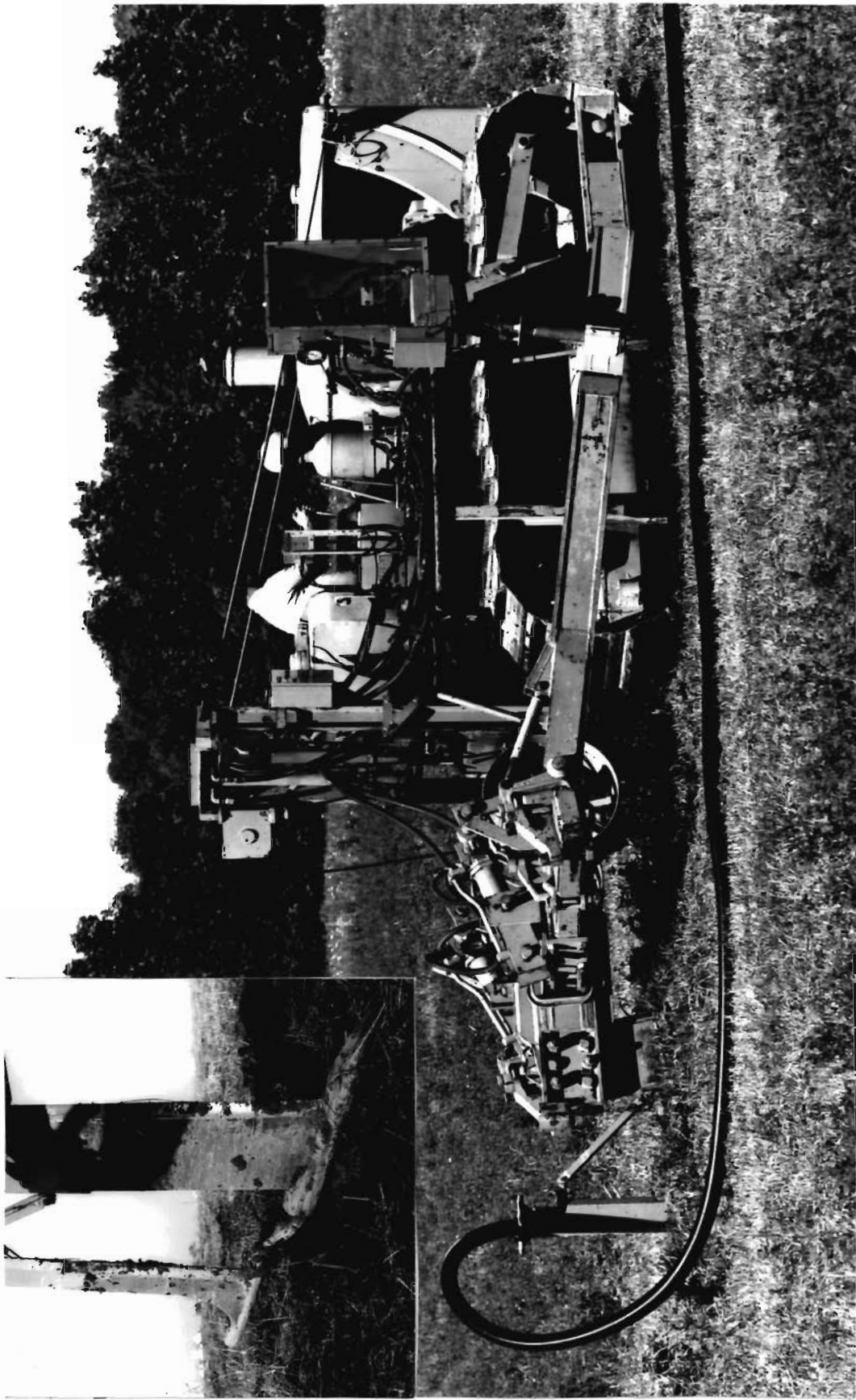


Fig. 2. USDA-ARS tool-bar-mounted, floating-beam mole plow adapted to install corrugated plastic drainage tubing.



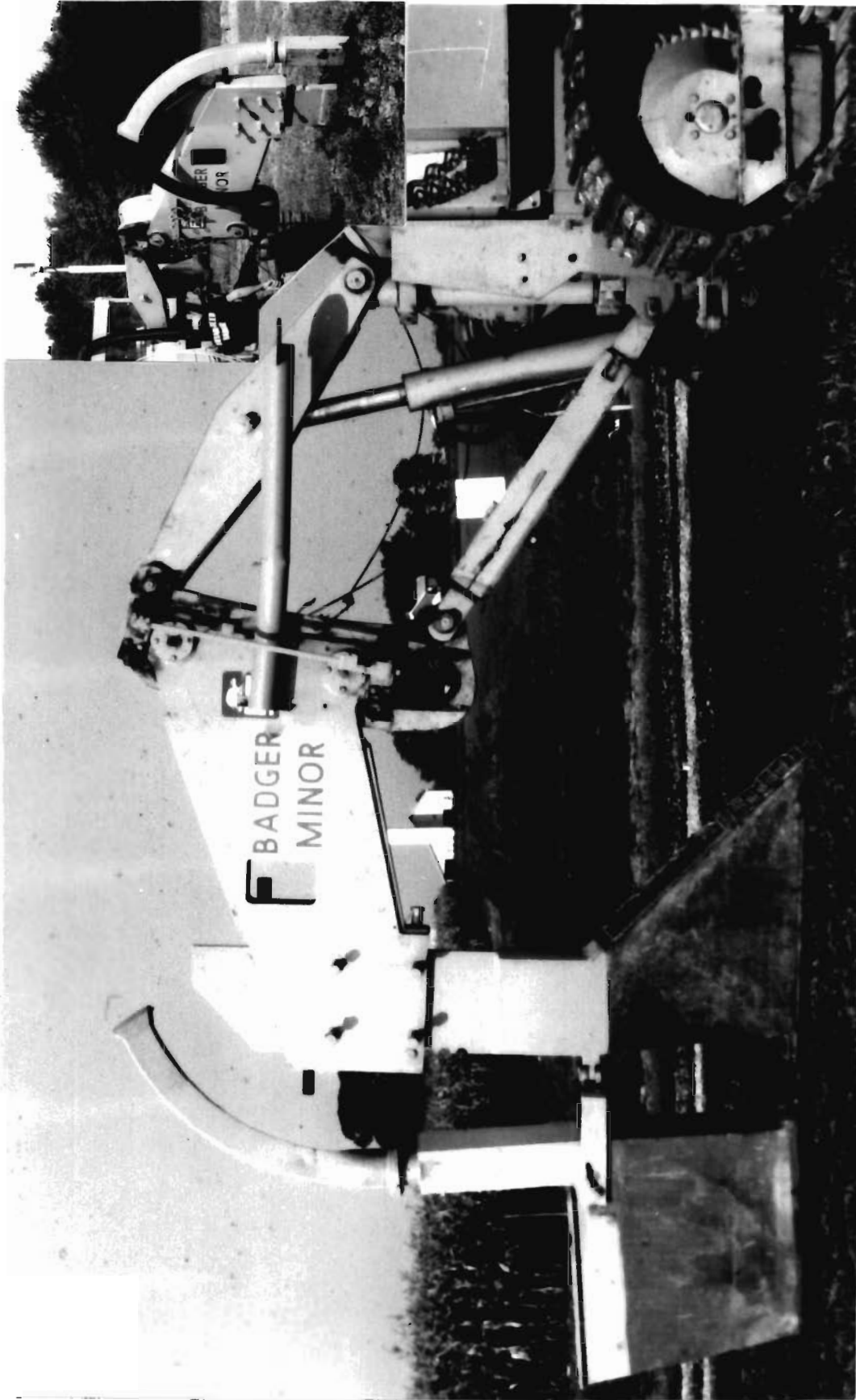


Fig. 3. Badger-Minor floating-blade drainplow equipped to install corrugated plastic drainage tubing (Photograph courtesy of Certain-Feed-Badger, Inc.).

stretched to the proposed grade; for examples, see Roe and Ayres (1954); "Asphalt pavers go electronic" (1962); and "Electronic controls on paver hold grade, crown, and slope" (1961). The time and labor required to set the wire reference are the major limitations. The idea of using a projected light beam dates back to the 1930's, when investigations were conducted by Sack (1933) in Germany. He used a conventional light source, collimated through a small diameter tube, which was aimed toward the operator seated on the drainage machine. Manual feedback control was employed.

Ede (1963, 1965) developed a semiautomatic optical-radio control system for his mole plow equipment. In this technique, the datum line was provided by the line-of-sight through an optical surveying level instrument, which was set up by a stationary observer behind the drainage machine. The observer could sight on a marker fixed to the plow blade and maintain the marker on the sight line directly by controlling the tractor hydraulic system by means of a switch and one-way radio link. Delayed reactions of the observer-controller and limited effective range were the major drawbacks.

Fouss, Holmes, and Schwab (1964) investigated the use of an "on-board", fluid-dampened pendulum device for "full" automatic control of a trenching machine and a floating-beam-type mole plow. In principle, it operated similar to an automatic leveling system in that it maintained the plow or trencher beam at a nearly constant slope (proportional to the desired drain gradient) independent of ground surface irregularities. This grade control device functioned quite well on the slow moving trenching machine, but on the high-speed mole plow it was not dependable

under many field conditions. Since the pendulum was an on-machine referencing device, its dynamic stability and its tendency to make accumulative errors were its major limitations.

In England, Gosling (1964) and Mathews and Harris (1964) developed a fully automatic grade and guidance control system based on a modulated light source datum. The equipment consisted of a tripod-mounted transmitter with a conventional 50-watt light source. The light was emitted with its beam axis on a chosen grade in the form of a "rotating fan" pattern of light. A photocell receiver fixed on the drainage machine received the pulsing light signal, which was frequency analyzed in a self-contained unit, so that the position of the sensor with reference to the center of the light beam was presented as an error signal. Feedback control via electrical hydraulic valves was used, or manual feedback was possible when the error signal was displayed on a meter. To this author's knowledge, this control system was never fully developed as a commercial unit.

Fouss and co-workers (1967, 1968a, 1968b) developed a working prototype of the laser beam automatic grade control system. The basic system components (Figure 4) consisted of: (a) A portable, tripod-mounted, low-powered laser beam projector to emit the light-line datum; and (b) a machine-mounted electronic receiver which adjusted the elevation of the plow hitch point hydraulically. The receiver was basically two closely spaced horizontal rows of phototubes which were protected from direct sunlight rays with a multi-baffled shadow box. If the receiver moved off the laser beam center, the imbalance between the phototube output of each row created an error signal for

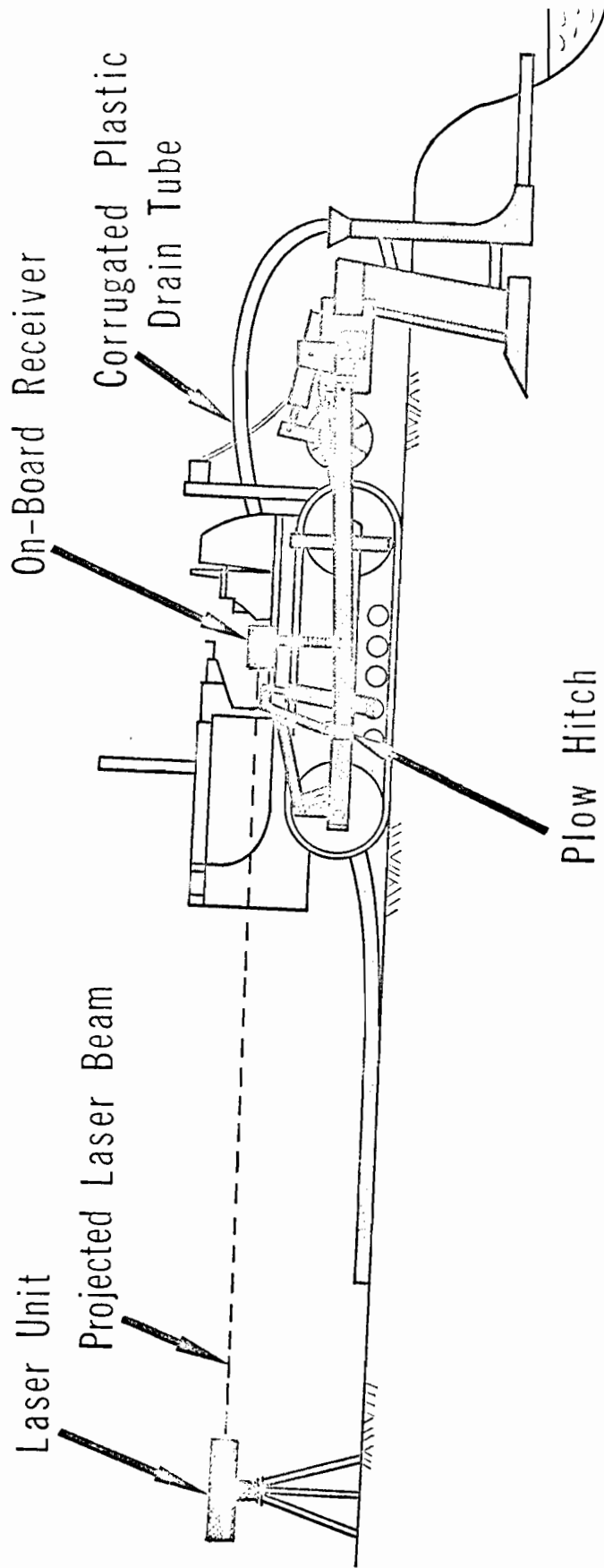


Fig. 4. Schematic of the laser-beam automatic depth and grade control system for the USDA-ARS draintube plow.

the operation of a solenoid hydraulic valve. The dead-zone of the receiver was adjustable from  $\pm 1/16$  inch to  $\pm 3/8$  inch. With a vertical hitch point velocity of 2.5 inches per second and the receiver mounted directly over the hitch, the receiver elevation could be maintained under most field conditions to within about  $\pm 1.0$  inch of the laser beam center.

A commercial version of an automatic laser control system for drainage equipment was manufactured by the Laserplane Corp. (1967). The "Laserplane" system is illustrated on a trenching machine in Figure 5. In this system, a thin "laser-plane" is generated as an elevation reference by a rotating the laser source--much like a light house beacon. The speed of rotation is 5 cycles per second. The laser-plane is tilted to the desired grade or slope at the source. The machine mounted receiver unit consists of a vertical array of photocells, each connected to a multivibrator in the controller circuit. The receiver-controller combination operates with a digital on-off mode of control; that is, each "sweep" of the laser beam past the receiver unit causes a corrective feedback motion to recenter the receiver on the laser-plane reference. Thus, the feedback motions occur in small steps, but the system is designed to cause larger feedback steps if the position error of the receiver is greater than  $\pm 1$  inch from the laser-plane. With this system, the reference plane is established over a large field area from one setup of the rotating source. Several machines could be simultaneously operated from the same source, if desired, such as when controlling land grading equipment, etc. One last feature of importance is the grade-change device, where the desired drain gradient is

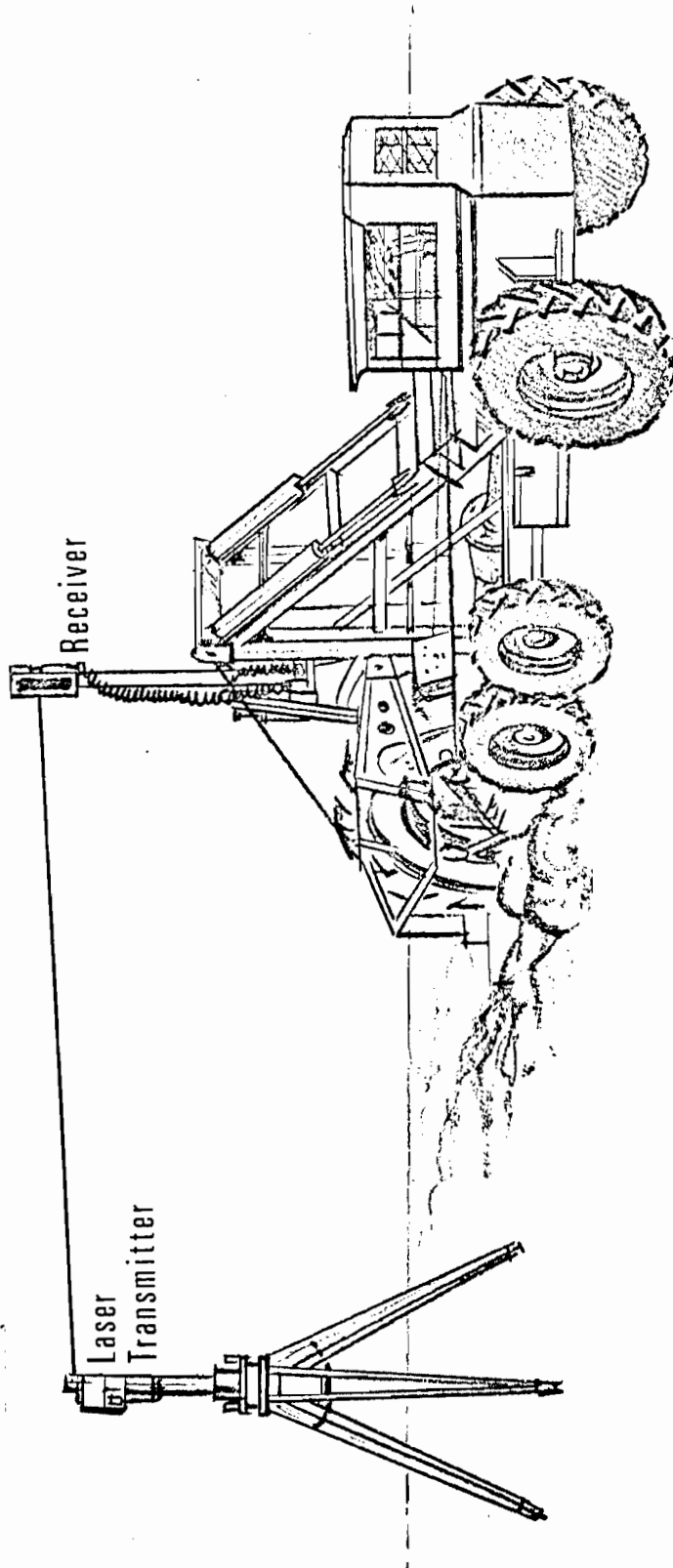


Fig. 5. The "Laserplane" grade control system on a tile trenching machine (Drawing courtesy of the Laserplane Corp.).

different than the slope to which the laserplane is set. This device causes the receiver unit to move vertically, relative to its machine mounting, as a function of ground travel. Thus, any desired grade can be created for a given laserplane slope. The use of this mechanism also permits changing the drain gradient at any point of travel along the drain line. This feature eliminates the need to reset the laserplane slope for each drain gradient change.

C. Soil Cutting with a Plow Blade

Nichols and Reaves (1958) measured the draft of subsoilers with several macroshapes of blades, including, normal-straight, inclined-straight, slightly curved, and exponentially curved (Figure 6). Draft for a 12-inch depth was reported for several soil types. The blade with the most curvature required the least draft, 7 to 20 percent less than the straight blade, in a highly compacted and cohesive soil. An inclined-straight blade was also reported to reduce draft nearly as much as the slightly curved blade. This conclusion was utilized in the design of the USDA-ARS mole plow (Figure 2) with a straight blade inclined 10 degrees from vertical. No effort was made by Nichols or Reaves to analytically relate blade shape to draft.

The cross-section of the plow blade, for example the leading edge, also can significantly affect draft. Gill and Vanden Berg (1967, p. 151) gave the reaction and friction forces on two blade cross-sectional shapes (Figure 7). Blade A has a leading edge wider than the shank, and its draft is less than for Blade B because there is no friction along its shank sides. Using equations developed by

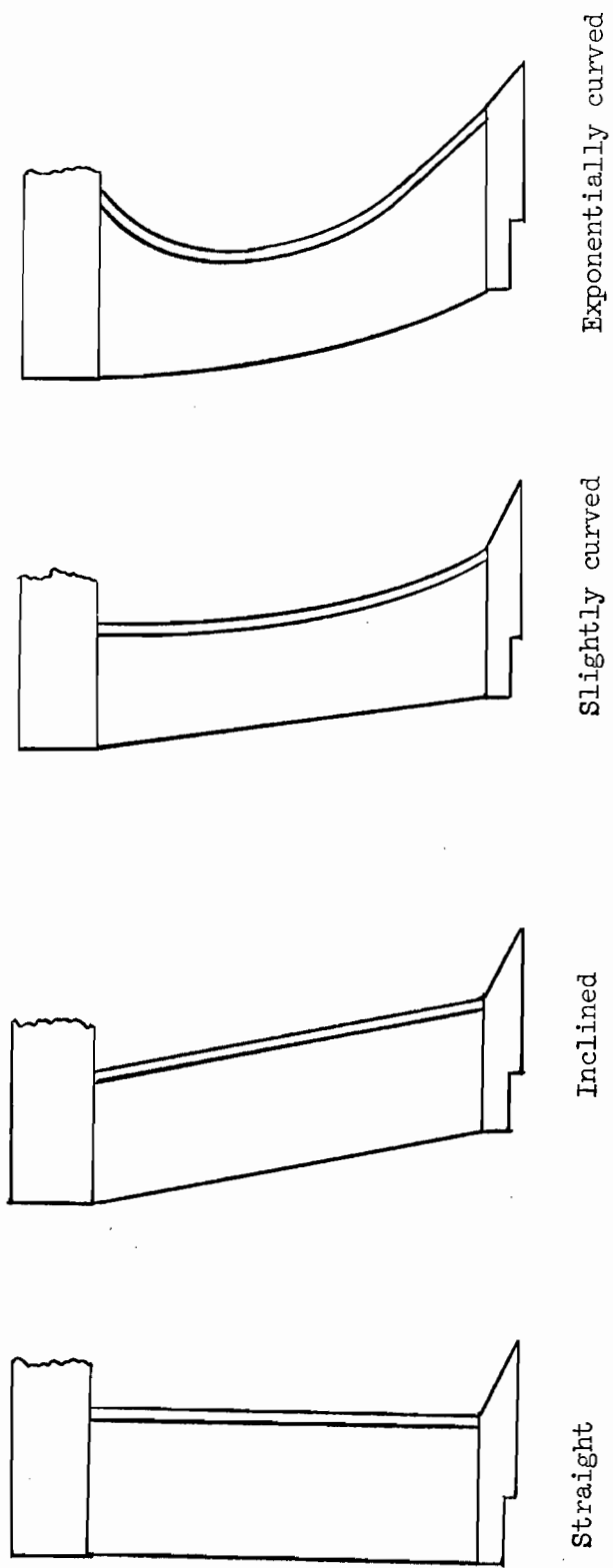
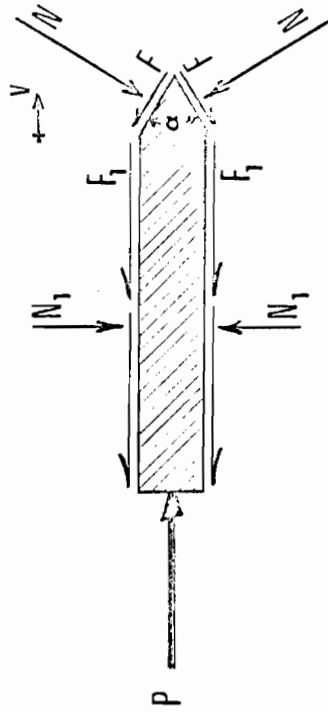
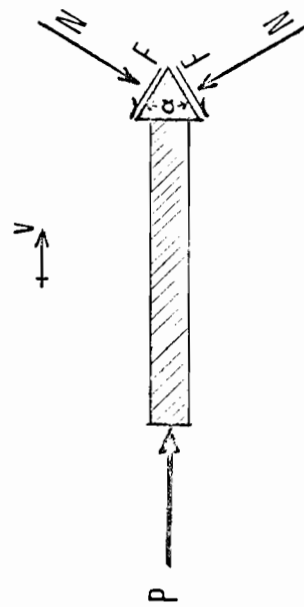


Fig. 6. Experimental subsoiler blade shapes [Nichols and Reaves (1958)]





Blade A



Blade B

Fig. 7. Blade cross-section versus soil cutting force (draft) [Gill and Vanden Berg (1967, p. 151)].

Kostritsyn (1956)<sup>1/</sup>, the draft for Blade B can be expressed as

$$P = 2N \sin \frac{\alpha}{2} + 2N \mu \cos \frac{\alpha}{2} + 2N_1 \mu .$$

The draft for Blade A is expressed in the same manner except the term  $(2N_1 \mu)$  for the side friction force is missing. The normal forces on the cutter wedge and sides were further defined as

$$N = K_1 A_1$$

where,

$K_1 \triangleq$  soil resistance to deformation,

$A_1 \triangleq$  surface area of the wedge,

and,

$$N_1 = K_2 A_2$$

where,

$K_2 \triangleq$  specific pressure of soil,

$A_2 \triangleq$  surface area of the side of the blade cutter.

Since the surface area of most plow blade sides is relatively large, it can be seen that draft can be reduced significantly if Blade A design is used rather than Blade B. This conclusion was also utilized in the design of the USDA-ARS mole plow (Figure 2) which has a cutter edge 3/8-inch wider than its shank.

Kostritsyn (1956)<sup>2/</sup> also studied soil cutting by vertical blades and the confinement effect on soil movement. He noted that near the ground surface, soil would rupture and move upward, whereas at greater

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<sup>1/</sup> Gill and Vanden Berg (1967, p. 152, eqs. 82, 83, and 84).

<sup>2/</sup> \_\_\_\_\_ (1967, pp. 149-150, fig. 96).

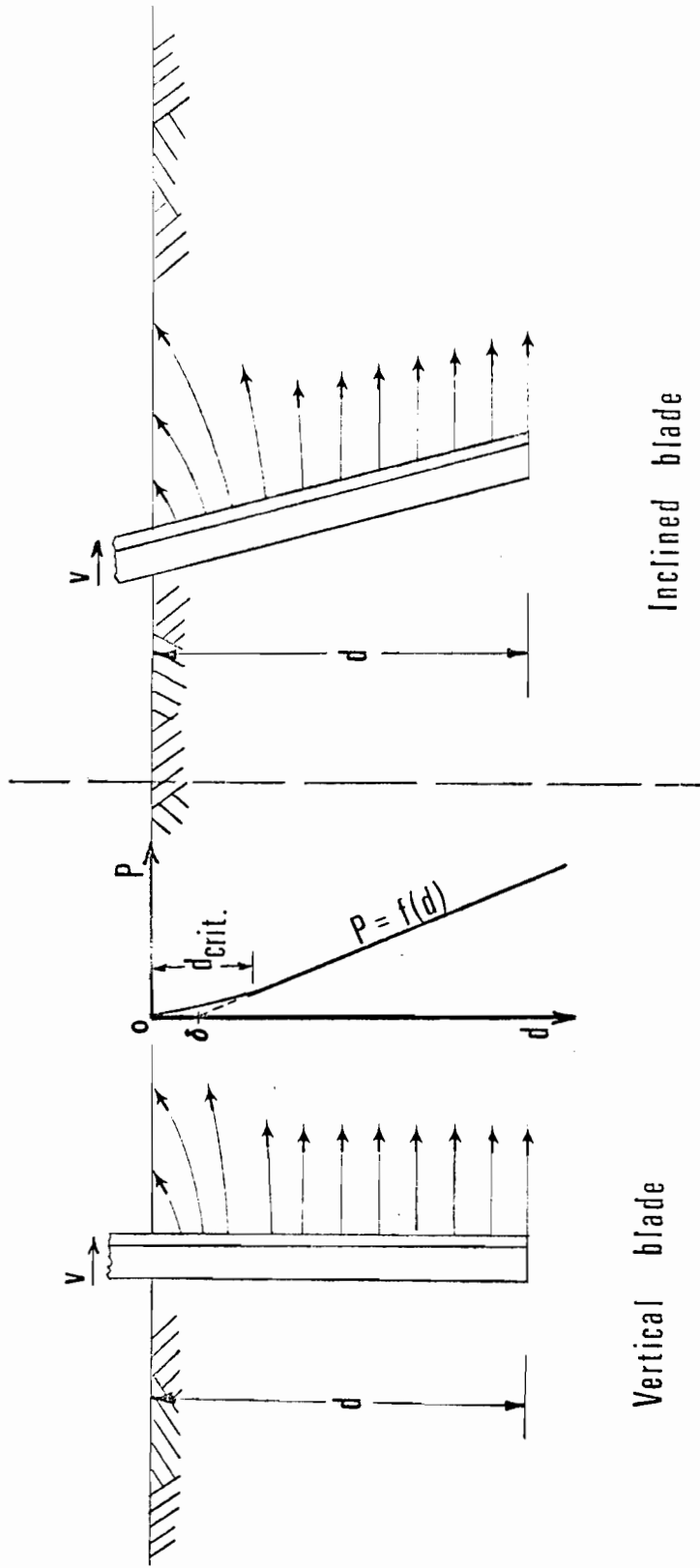
depth soil movement was parallel to the direction of travel of the cutter. One might extend this concept to the inclined blade (Figure 8) and conclude that the depth where soil movement is parallel to the cutter travel would probably be greater than for the vertical cutter. Kostritsyn measured draft  $P$  of the vertical cutter versus the depth of operation  $d$  in a uniform soil and found that below a critical depth of 20 to 25 cm. for a 3 cm. thick blade, a nearly linear relationship existed between draft and depth (Figure 8). This would imply that below the critical depth, the soil pressure  $p_s$  on the cutting edge was essentially constant. If one neglects the pressure  $p_s$  above a small depth  $\delta$  (Figure 8), one can approximate the draft force  $P$  as follows:

$$P = w p_s (d - \delta) ,$$

where  $w$  is the blade thickness. This analysis further implies that the draft force  $P$  acts at  $[\delta + \frac{1}{2} (d - \delta)]$  below the ground surface, which is approximately  $(\frac{1}{2} d)$  if  $\delta$  is much smaller than  $d$ . However, soil pressure on the blade changes with depth for most soils. If one assumes that it is proportional to depth, that is,  $p_s = K_s d$ , where  $K_s$  is the rate of change in lb./ft.<sup>3</sup>, one gets

$$P = \frac{1}{2} w K_s d^2 .$$

The line-of-action for this draft force is  $(\frac{2}{3} d)$  below the soil surface. It is noted that in both of the above examples, the line-of-action of the draft (soil resistance force) on the cutting blade is a constant fraction of operating depth.



[Gill and Vanden Berg (1967, p. 150, fig. 96)]

Fig. 8. Soil movement by vertical and inclined cutting blades.

$$P = K d^p$$

where,

$P \triangleq$  cutting force (draft),

$K \triangleq$  coefficient of soil resistance,

$d \triangleq$  depth of operation,

$p \triangleq$  coefficient,

has been found to hold for several types of both vertical and horizontal soil cutters, and different soil types. Gill and Vanden Berg (1967, p. 189) reported that Zelenin, a Russian investigator, found the exponent  $p$  to be nearly constant at 1.35 for horizontal cutters. However, studies made by Reaves at the National Tillage Machinery Laboratory<sup>1/</sup> revealed that, "...the exponent [ $p$ ]....may vary considerably with depth and soil type."

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<sup>1/</sup> Gill and Vanden Berg (1967, p. 190).

## II. SIMULATION OF A MOLE PLOW

The first objective of this study was to develop a mathematical model to simulate the dynamic response of plow-type drainage equipment pulled across a field by a crawler tractor. A floating-beam mole plow was selected for analysis since recent investigations have indicated the potential of adopting such a plow for installing corrugated plastic drainage tubing [Fouss (1968)].

### A. Mathematical Model

A simplified model of the mole plow and the forces acting on it are shown in Figure 9. The following simplifications were made in the analysis:

- (1) The hitch point H is a frictionless hinge and the plow beam and blade assembly HBM is rigid;
- (2) Ground surface irregularities from the 'mean ground surface reference plane' have an insignificant effect on the soil resistance R;
- (3) The plow is pulled forward at a constant ground speed  $v$ ;
- (4) The depth of the intersection of the soil resistance R with the vertical line BM, a distance  $b$  behind the hitch point, is a constant fraction  $r$  of the moling depth  $d$ ;
- (5) For a given soil and a constant ground speed, the horizontal resistance component  $R_H$  (draft) is a function of

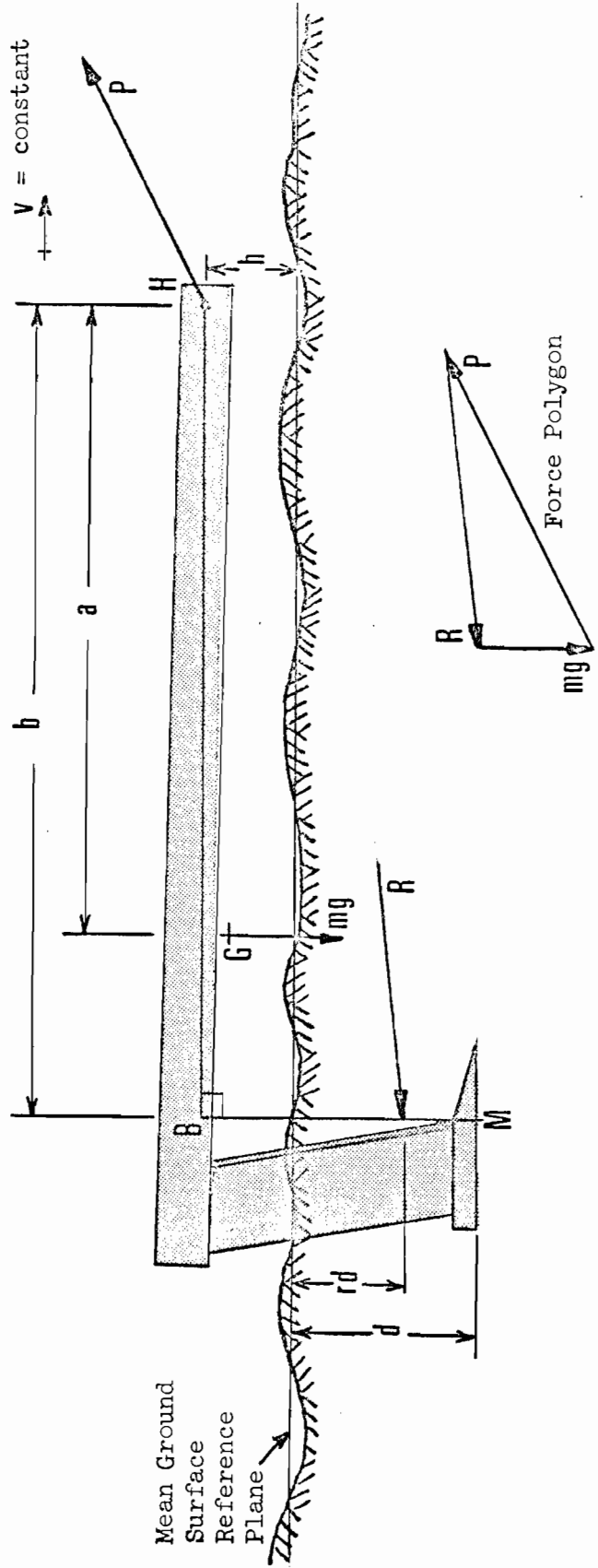


Fig. 9. Simplified model of a mole plow.

the moling depth  $d$  only,

$$R_H \triangleq R_H(d) \dots \dots \dots [1]$$

The vertical component  $R_V$  consists of a component dependent upon the moling depth, and another component proportional to its time-rate of change  $\dot{d}$ ,

$$R_V \triangleq f_V(d) - C \dot{d} \dots \dots \dots [2]$$

where  $C$  is a damping coefficient.

A free-body diagram of the plow model is shown in Figure 10.

The equations describing the plow motion were derived by applying D'Alembert's Principle as follows:

Summation of horizontal forces (no inertia force,  $v = \text{constant}$ ) yields:

$$P_H - R_H = 0 \dots \dots \dots [3]$$

Summation of vertical forces yields:

$$P_V - R_V - mg - m(\ddot{h} + a\ddot{\theta}) = 0 \dots \dots \dots [4]$$

Summation of moments about the plow hitch point H yields:

$$(rd + h) R_H - bR_V - amg - am(\ddot{h} + a\ddot{\theta}) - J_G \ddot{\theta} = 0 \dots \dots \dots [5]$$

Noting that the plow angular movements are quite small,  $\tan \theta \approx \theta$  and  $\cos \theta \approx 1$ ; hence,  $y \approx y_0$  (Figure 10), and



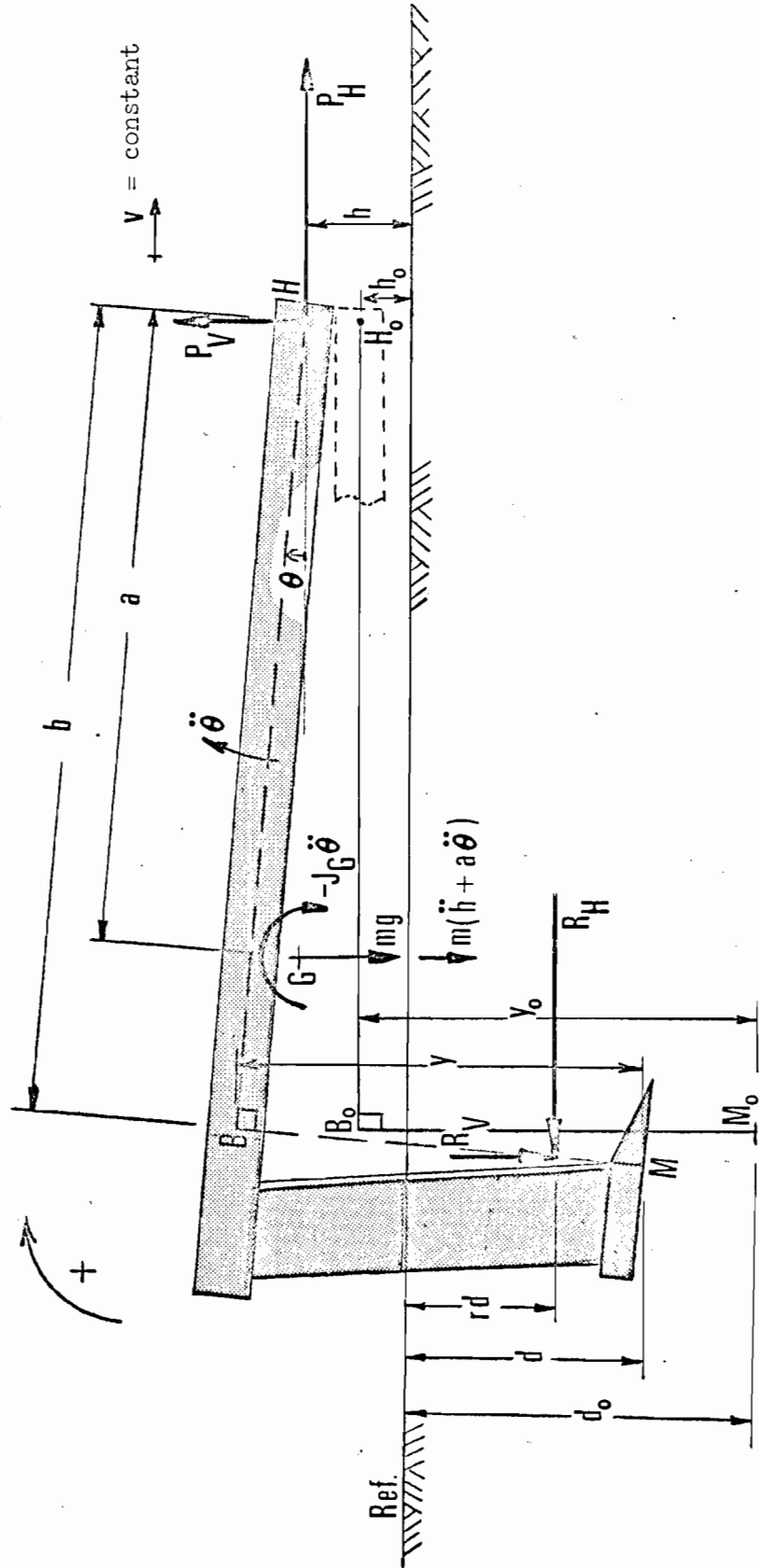


Fig. 10. Dynamic free-body diagram of mole plow model.

$$\theta \approx \frac{y - d - h}{b} , \text{ in radians,}$$

and

$$\ddot{\theta} = - \frac{\ddot{d} + \ddot{h}}{b} \dots \dots \dots [6]$$

The steady-state component of  $R_V$  for a constant, steady-state hitch height  $h_S$ , can be expressed in terms of  $R_H$ , by substituting

$\ddot{h} = 0$ , and  $\ddot{\theta} = 0$  into equation [5], and rearranging:

$$f_V(d) = \frac{(r d_S + h_S)}{b} R_H - \frac{a}{b} mg , \dots \dots \dots [7]$$

where,  $d_S$  is the steady-state depth corresponding to  $h_S$ . At this point, it is assumed that the coefficient  $(r d_S + h_S)$  is nearly constant, that is, the steady-state  $R_H$  acts at a constant distance below  $H$ , for a fixed plow configuration and relatively small changes in  $d$ . <sup>1/</sup>

Substituting equation [7] into equation [2], one gets

$$R_V = \frac{n}{b} R_H - \frac{a}{b} mg - C \dot{d} , \dots \dots \dots [8]$$

where,

$$n \triangleq (r d_S + h_S) \dots \dots \dots [9]$$

Using equations [6] and [8], equations [3], [4], and [5] reduce to the following model which describes the mole plow dynamic response:

$$P_H = R_H \dots \dots \dots [10]$$

$$P_V = (1 - \frac{a}{b}) m (\ddot{h} + g) - \frac{a}{b} m \ddot{d} - C \dot{d} + \frac{n}{b} R_H \dots \dots [11]$$

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<sup>1/</sup> This assumption is discussed in detail on page 28 .

$$\ddot{d} + \left(\frac{b^2 c}{J_H}\right) \dot{d} + \left(\frac{b(rd + h - n)}{J_H}\right) R_H = \left(\frac{a b m}{J_H} - 1\right) \ddot{h}, [12]$$

where,

$$J_H \triangleq J_G + a^2 m \dots \dots \dots [13]$$

B. System Parameters

Since equation [12] is nonlinear, the usual methods of identifying the system dynamic response parameters are not directly applicable. However, one can obtain some insight into the system behavior if it is linearized. Assuming that the draft force can be approximated as a power function of the moling depth:

$$R_H = K d^p, \dots \dots \dots [14]$$

where K and p are constants (will be discussed in more detail later), equation [12] can be linearized by considering the plow response for a step change in hitch height, from  $h_0$  to  $h_s$  ( $\dot{h}_0 = \dot{h}_s = 0$ ), as follows:

$$\ddot{d} + \left(\frac{b^2 c}{J_H}\right) \dot{d} + \left(\frac{b K (r d_0 + h_s - n)(d_0)^{p-1}}{J_H}\right) d = 0, [15]$$

where  $d_0$  and  $d_s$  correspond to  $h_0$  and  $h_s$ , respectively. This is the equation of a linear second-order system. Its natural frequency is given by

$$\omega_n = \sqrt{\frac{b K (r d_0 + h_s - n)(d_0)^{p-1}}{J_H}} \dots \dots \dots [16]$$

It is noted that as  $d \rightarrow d_s$ ,  $\omega_n \rightarrow 0$ . One might consider the d-coefficient,  $\omega_n^2$ , as a 'variable gain factor' whose magnitude approaches zero as equilibrium is reached. As  $\omega_n \rightarrow 0$ , the plow loses its "tendency" to oscillate.

Equation [15] also indicates that the plow damping is provided by the damping term associated with  $R_V$  (equation [2]). Experience has shown that for a "long beam" mole plow, the depth of penetration does not oscillate, therefore, the plow must be at least critically damped (that is,  $\zeta \geq 1$ ). The critical damping coefficient  $C_c$  of equation [2] would be given by

$$C_c = \frac{2 J_H \omega_n}{b^2} , \dots \dots \dots [17]$$

where  $\omega_n$  is given by equation [16]. If one estimates the plow damping to be 5 times critical, for example, the value of  $C$  is set at

$$C = 5 C_c \dots \dots \dots [18]$$

### C. Simulation

The USDA-ARS tool-bar mounted, floating-beam mole plow (Figs. 2 and 4) was selected for simulation and study on an analog computer. The basic dimensions and parameters of the mole plow are given in Table 1. A more complete description of the plow is given in Appendix A. The value of the constant fraction  $r$ , and the magnitude of  $n$  for various mole-hitch configurations, were determined by field testing the prototype plow (Appendix B); the data confirmed the linear steady-state relationship given by equation [9] for a significant range of moling depth. In the computer simulation, a typical value of  $n = 2.817$  ft. was used, and the field data indicated that it was applicable for at least the range,  $1.5 \leq d \leq 3$  ft. (Fig. 49, p.99). Also, data were collected from which a draft equation was developed for  $1 \leq d \leq 3$  ft. (Appendix C):

$$R_{11} = 3400 d^{1.65} \dots \dots \dots [19]$$

The draft versus depth relationship for  $d < 1$  ft. was approximated by assuming the minimum draft to be equal to the sliding friction force (estimated at 1500 lb.) between the mole M and the ground surface (Appendix C, p. 126).

TABLE 1. Dimensions and Physical Parameters for USDA-ARS Tool-Bar Mounted, Floating-Beam Mole Plow

$a = 8.92$ ft.	$m = 124.2$ lb.-sec. <sup>2</sup> /ft.
$b = 11.5$ ft.	$J_G = 1,458$ ft.-lb.-sec. <sup>2</sup>
$n = 2.817$ ft.	$J_H = 11,340$ ft.-lb.-sec. <sup>2</sup>
$r = 0.8$	$\dot{h}_c = \pm 0.20$ ft./sec. (Maximum)
$0 \leq d \leq 3$ ft.	$0.5 \leq h \leq 2.5$ ft.

The plow model described by equations [10], [11], and [12] was programmed on an E.A.I. TR-48 analog computer using the dimensions and parameters given in Table 1. A constant ground speed  $v$  of 2 ft./sec. was assumed in the study. The amplitude-scaled and real-time computer circuit is shown in Figure 11. The soil resistance  $R_H$  relationship was set on a variable function generator (Appendix C, Fig. 65). The excitation for this model is the hitch acceleration  $\ddot{h}$  and/or initial displacement  $h_i$ . An excitation circuit was designed to simulate the hitch acceleration pulses when the hitch point H is moved by the hydraulic cylinders (Appendix D, Fig. 66).

#### 1. Setting damping coefficient C

The first step in the simulation study was to estimate the damping

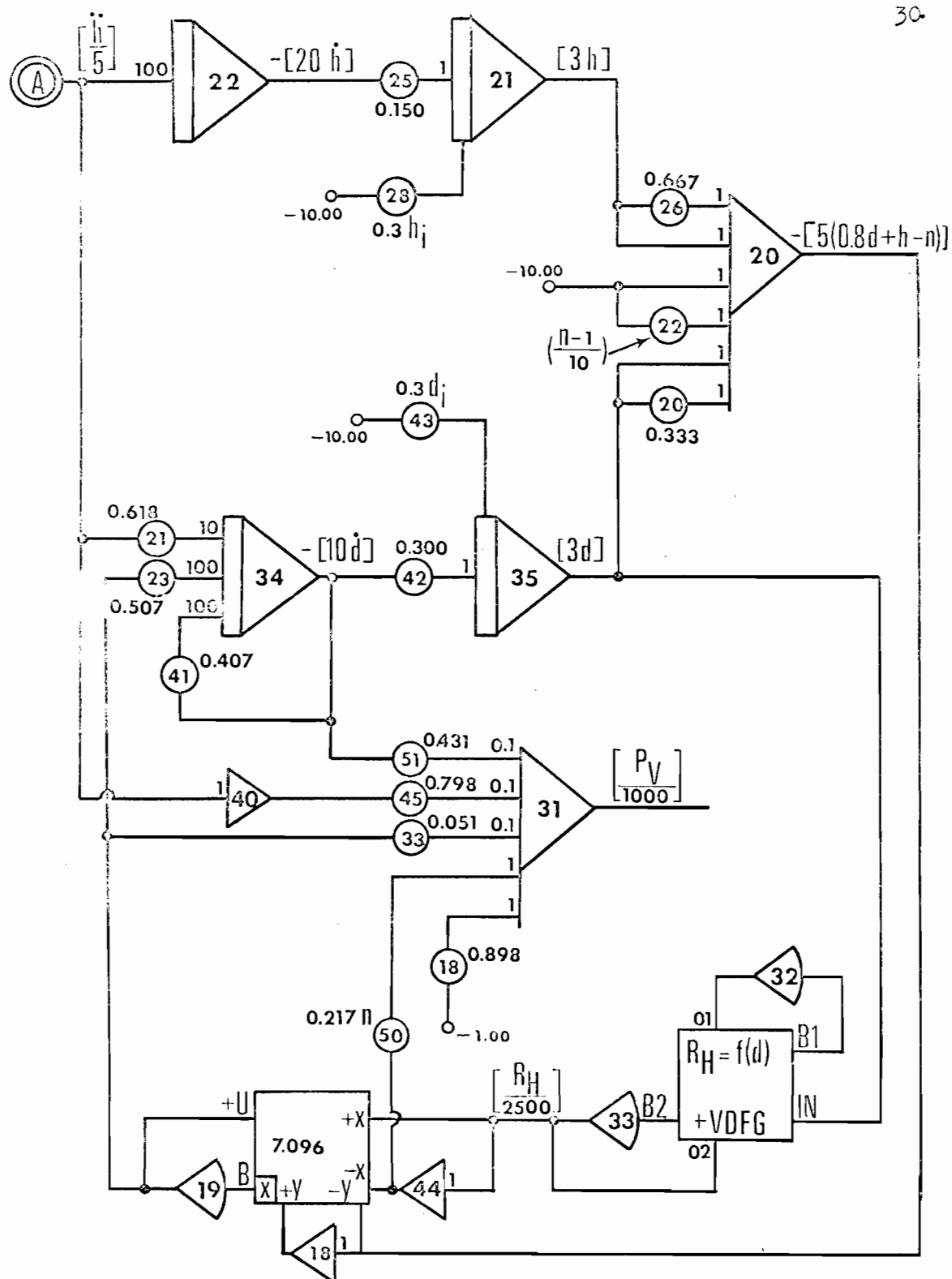


Fig. 11. Analog computer circuit for simulation of mole plow dynamic responses.

coefficient  $C$ . An initial estimate of 3215 lb.-sec./ft. was determined by the use of equations [16] to [18], assuming  $d_0 = 3$  ft., and  $(r d_0 + h_s - n) = 2$  ft.; that is,  $h_0 - h_s = 2$  ft., which would cause a very large unbalanced moment about the hitch. The effective natural frequency of the plow for this unbalance (equation [16]) is 3.75 rad./sec. (0.6 cps). By trial-and-error, the computer closely simulated field tests 2, 4, and 5 (Appendix B) as shown in Figures 12, 13, and 14, respectively, for a damping coefficient  $C = 3490$  lb.-sec./ft. The plow mole  $M$  is at a steady-state operating position for the indicated 0 ft. of ground travel, and in dynamic response at  $0^+$  ft.

## 2. Effect of hitch acceleration $\ddot{h}$

The simulated hitch-point motion by the hydraulic depth adjusting cylinders is shown in Figure 15. The effects of the hitch acceleration and deceleration pulses on vertical draft component  $P_V$  and the moling depth time-rate of change  $\dot{d}$  is shown in Figure 16. Similar computer runs were made with the  $\ddot{h}$ -term neglected and  $\dot{h}$  assumed a pure step function and the computed mole depth  $d$  could not be distinguished from the previous run. The only difference was the absence of the pulses at the beginning and end of the hitch motion in the plots of  $\dot{d}$  and  $P_V$ .

The effect of a 0.3 ft. "free-fall" of the plow hitch point ( $\ddot{h} = -g$ ) is shown in Figure 17. The circuit for implementing the free-fall condition on the computer is given in Appendix D, Fig. 67. It is noted in Figure 17 that the free-fall of the hitch causes a momentary decrease in the vertical component of pull  $P_V$ , which in turn results in a small momentary decrease (0.01 ft.) in plowing depth. The free-

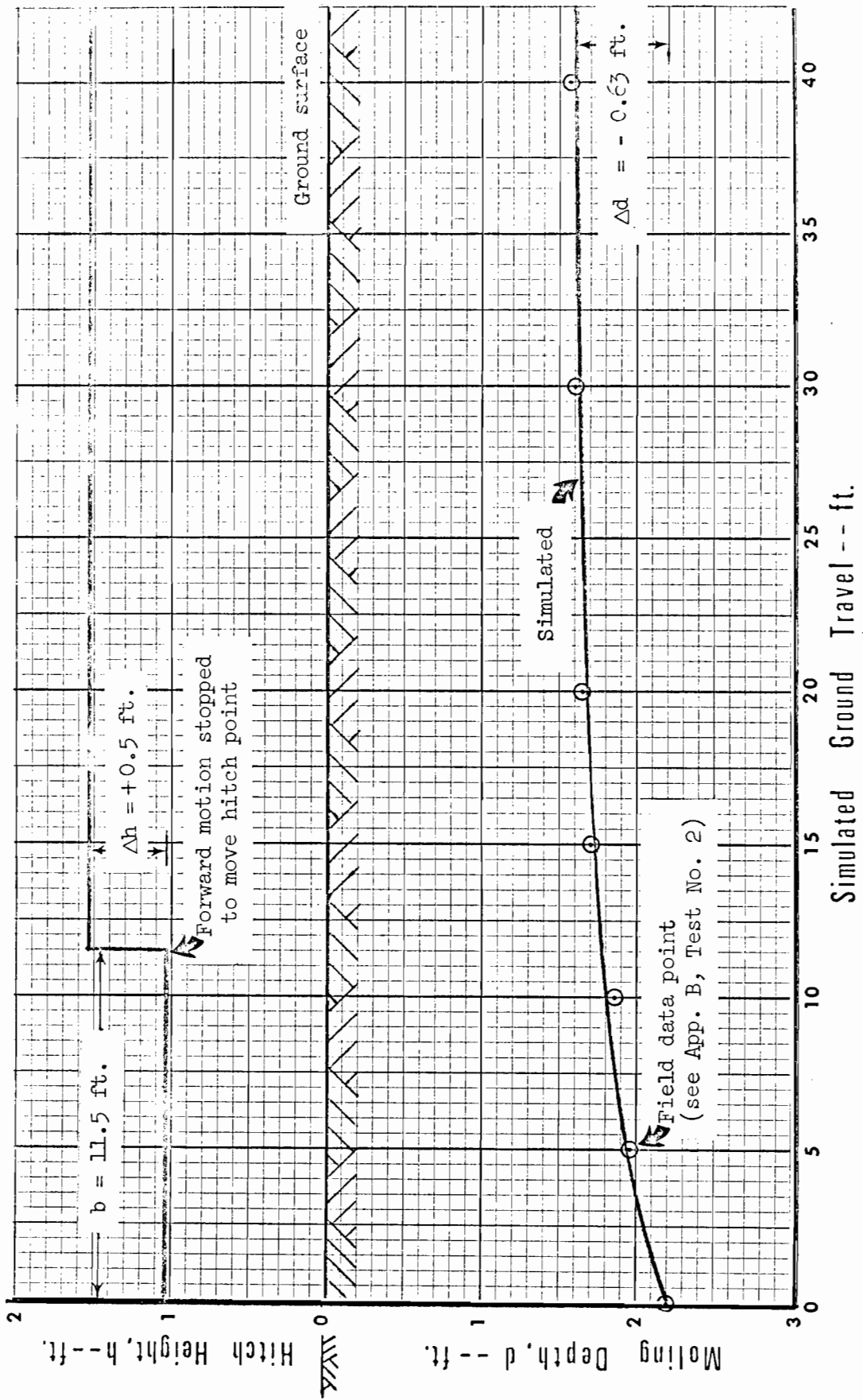


Fig. 12. Comparison of simulated and actual mole plow response to an upward step displacement of the hitch.



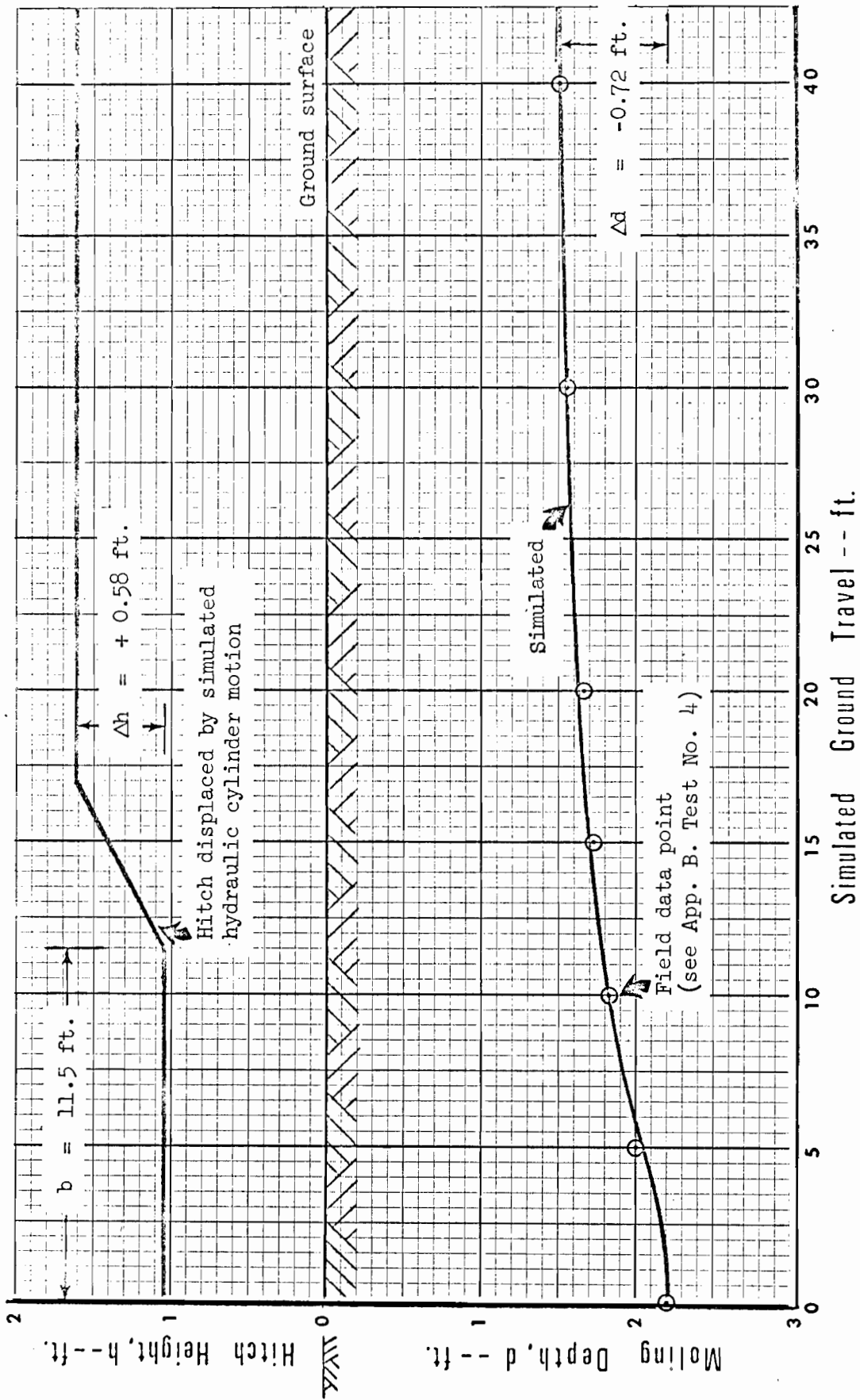


Fig. 13. Comparison of simulated and actual mole plow response to an upward ramp-step hitch displacement.

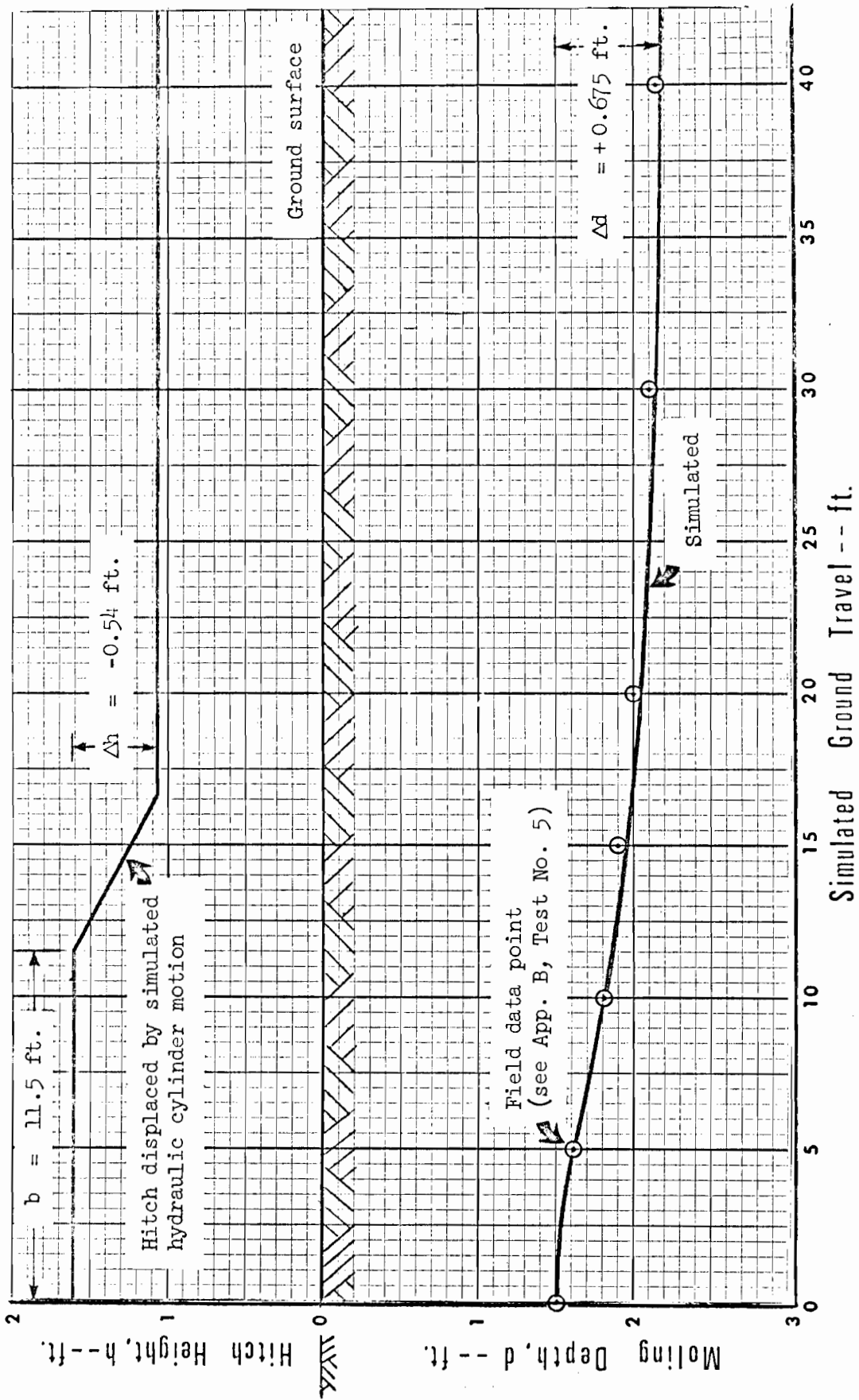


Fig. 14. Comparison of simulated and actual mole plow response to a downward ramp-step hitch displacement.

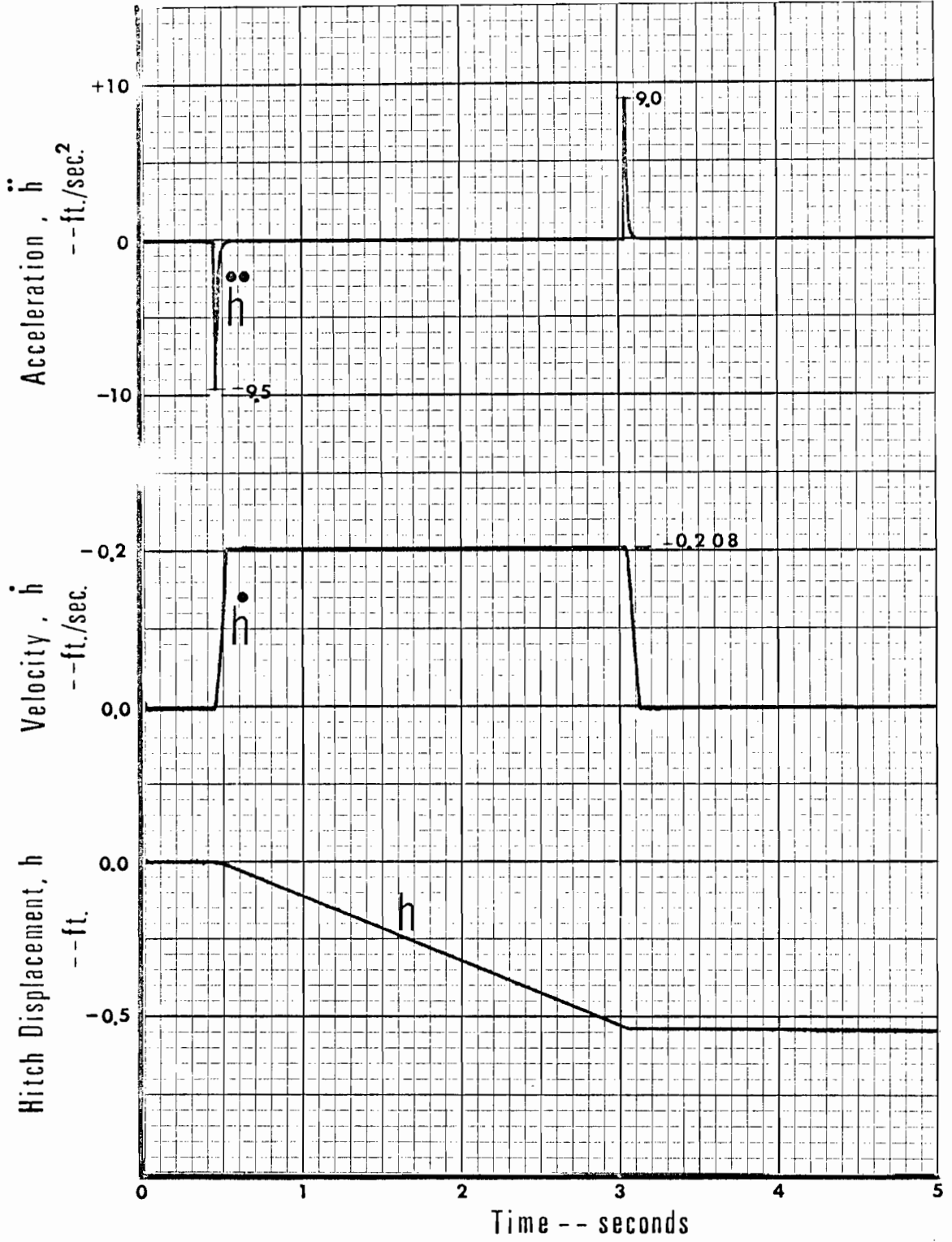


Fig. 15. Acceleration, velocity, and displacement of simulated hitch motion by the hydraulic depth adjusting cylinders.

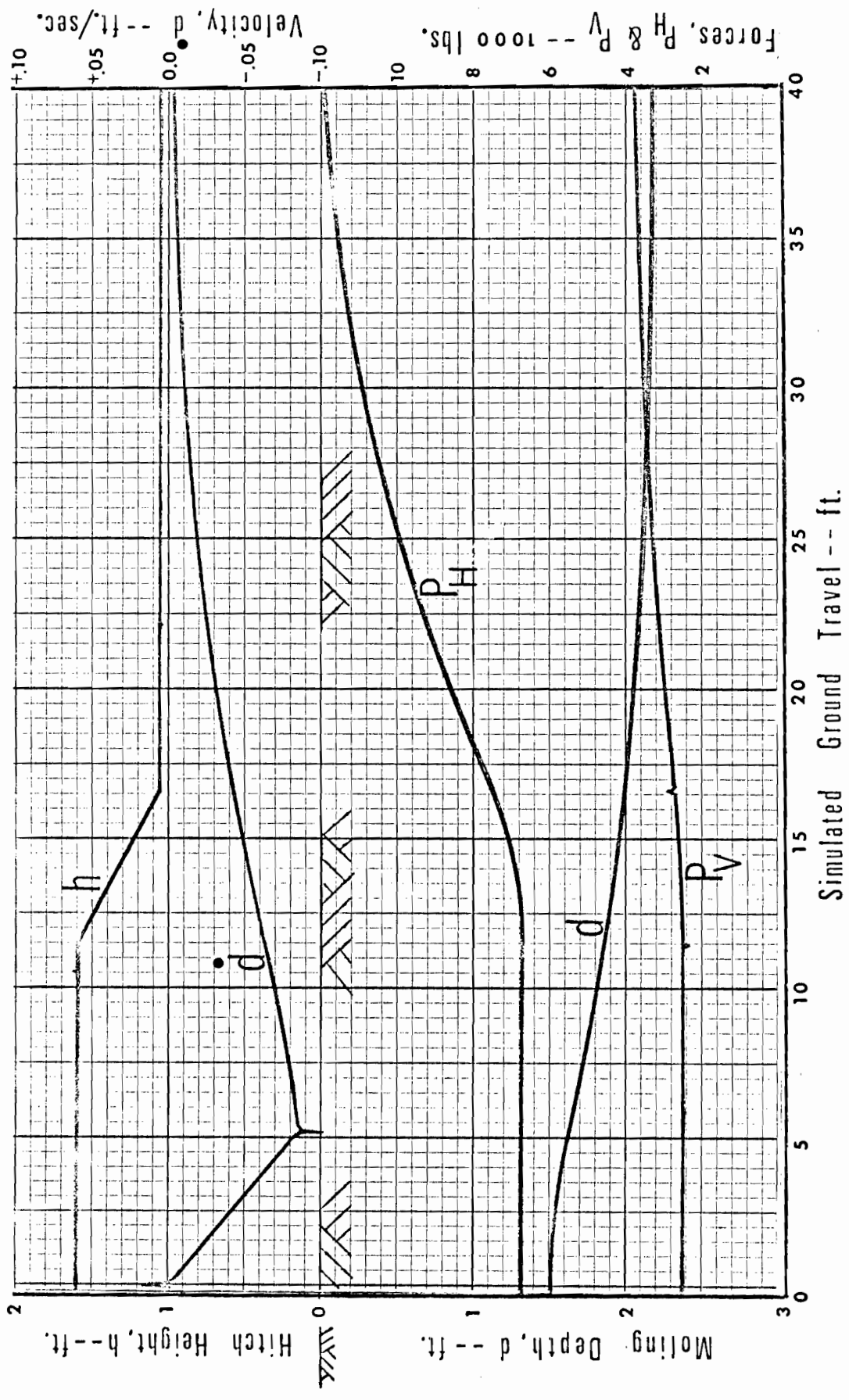


Fig. 16. Simulated mole plow response showing the effects of hitch acceleration.

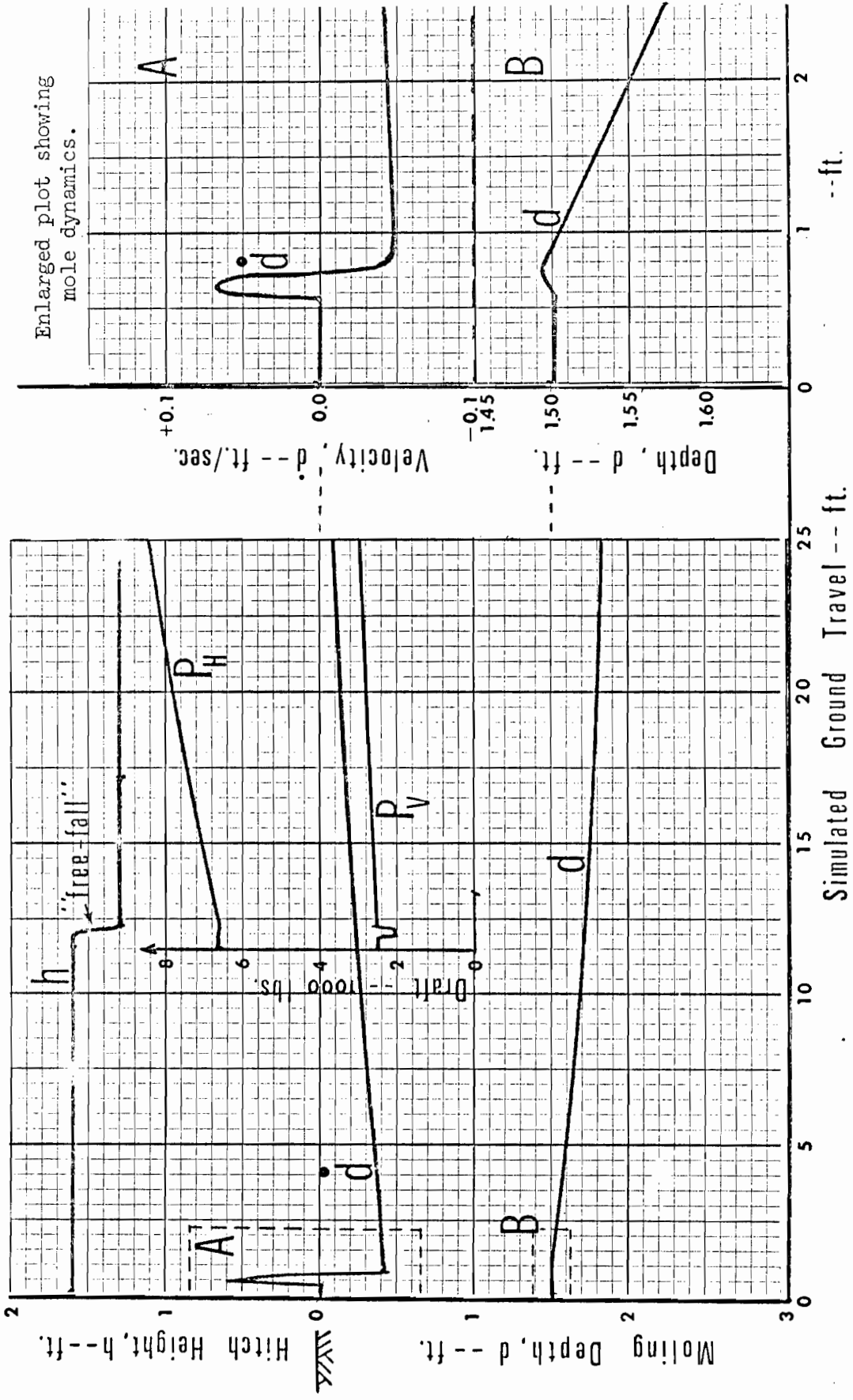


Fig. 17. Simulated "free-fall" of the plow hitch point.

fall case may not occur for the prototype plow but it might be closely approximated when the crawler tractor tracks "go over center" on uneven ground surfaces, thus causing the hitch to drop rapidly.

### 3. Settling time (distance) for step inputs

The response of the plow to step changes in the hitch position gives an indication of its speed-of-response. One can express the speed-of-response in terms of the "10 percent settling time"<sup>1/</sup> (or distance of ground travel for the plow); that is, the ground travel required, following the change in hitch height, for the corresponding change in moling depth to be 90 percent completed and 10 percent remaining to reach equilibrium. Since the model is nonlinear, it is necessary to specify the 'settling distance' for different magnitudes of step changes in the hitch position and for different depths of operation. The simulated response of the plow to step changes<sup>2/</sup> in the hitch position is shown in Figures 18 and 19 for moling depths of 2.0 ft. and 2.5 ft., respectively. It can be seen that the mole can penetrate to a greater depth faster than it can decrease in operating depth. In addition, the speed of response increases with increasing depth of moling. The 10 percent settling distances given in Table 2 were estimated from Figures 18 and 19.

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<sup>1/</sup> Doebelin, E. O. "Dynamic Analysis and Feedback Control," McGraw-Hill, N. Y., 1962, p. 161, fig. 5.2.

<sup>2/</sup> The step changes were implemented on the computer as initial conditions, which is analogous to moving the hitch point while forward motion is stopped.

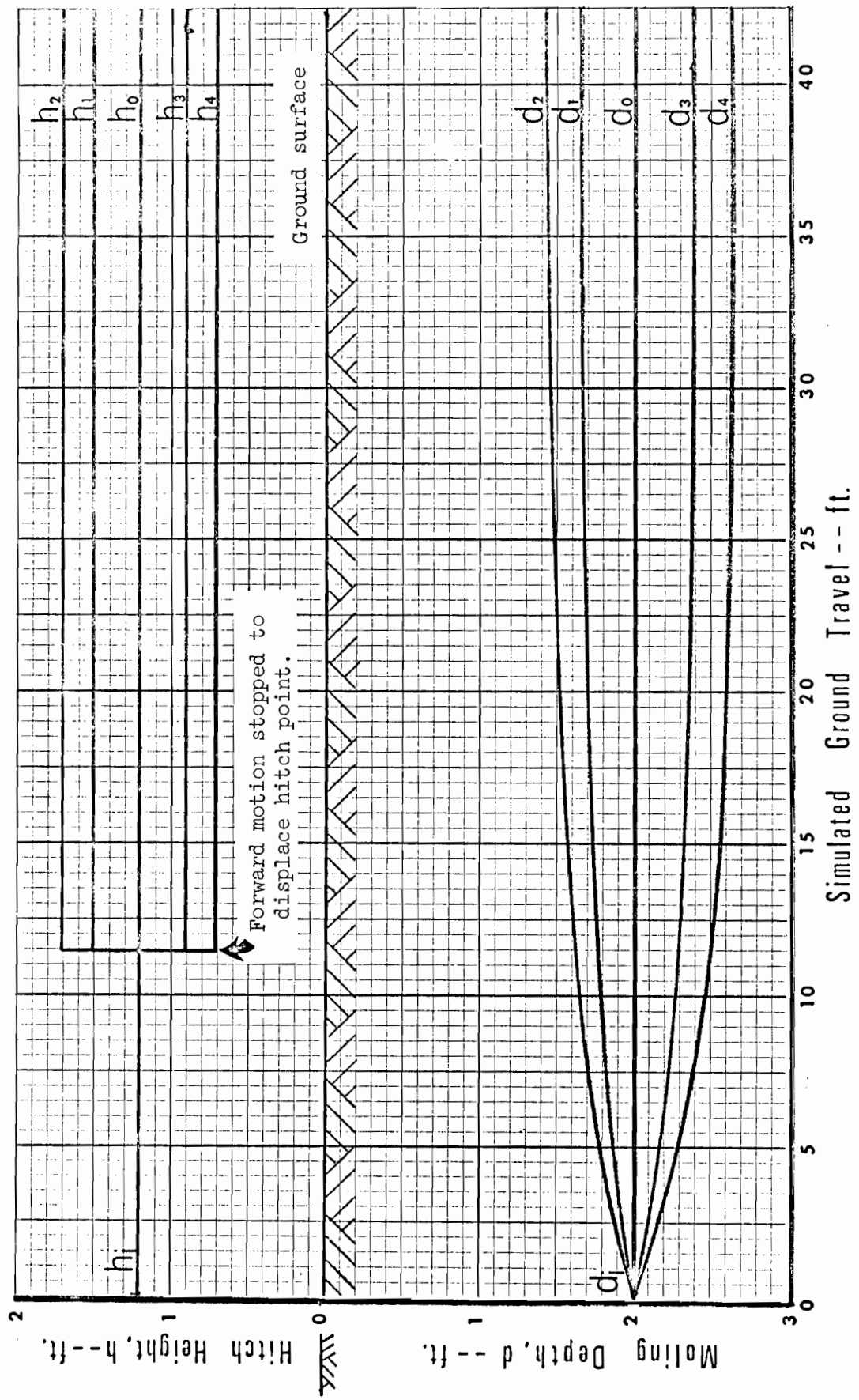


Fig. 18. Simulated response of the mole plow to step changes in hitch height;  $d_i = 2.0$  ft.

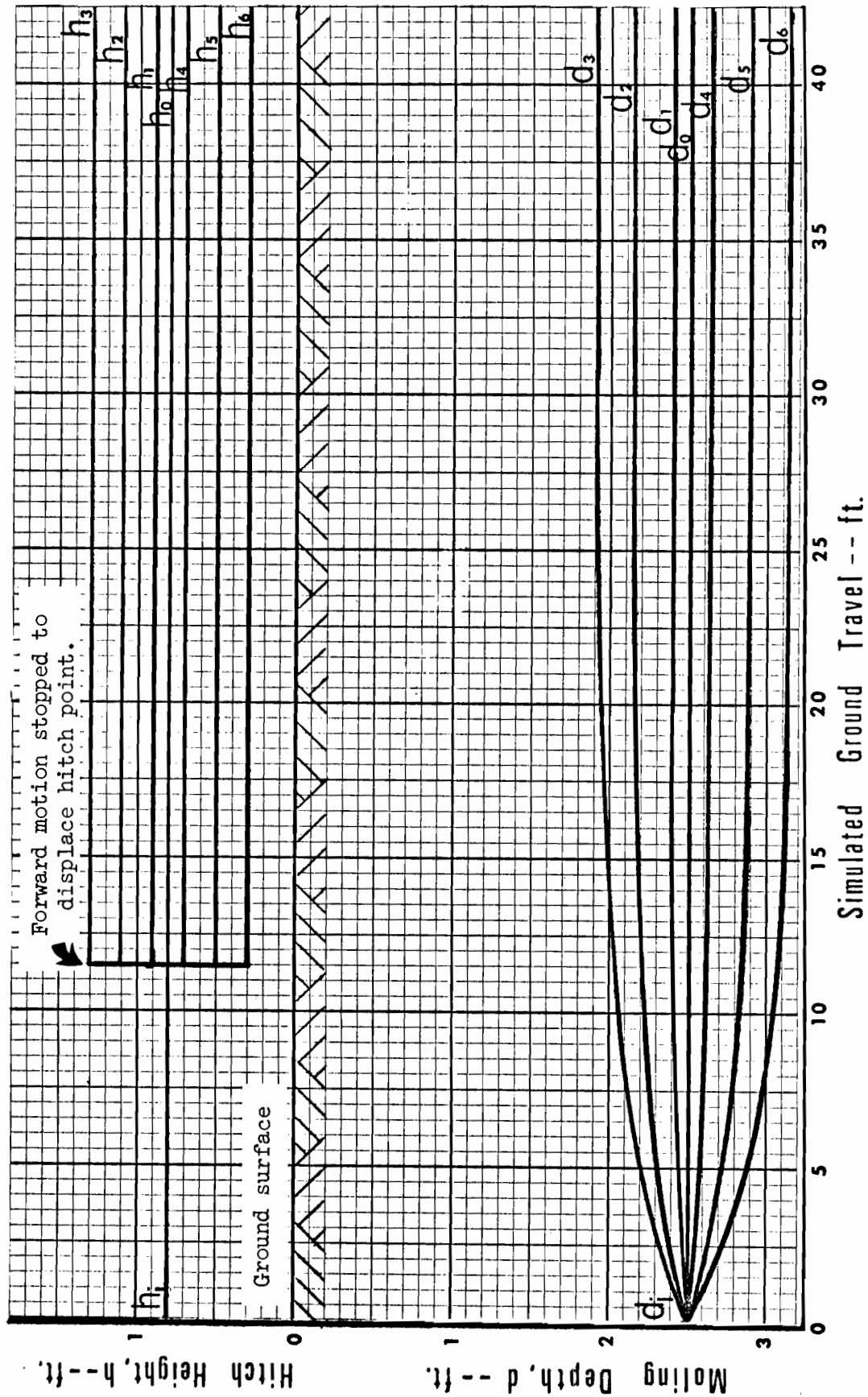


Fig. 19. Simulated response of the mole plow to step changes in hitch height;  $d_i = 2.5$  ft.



TABLE 2. Ten Percent Settling Distances for Molding Depth Following Step Changes in Hitch Height

$d_i$ (ft.)	$h_i$ (ft.)	$h_f - h_i$ (ft.)	$d_f$ (ft.) <sup>1/</sup>	$d_{10\%}$ (ft.) <sup>2/</sup>	Settling Distance (ground travel) at $d_{10\%}$ (ft.)
2.00	1.217	+0.5	1.375	1.44	33
-do-	-do-	+0.3	1.625	1.66	33.5
-do-	-do-	-0.3	2.375	2.34	17
-do-	-do-	-0.5	2.625	2.56	15
2.50	0.817	+0.5	1.875	3.06	21.5
-do-	-do-	+0.3	2.125	2.84	23
-do-	-do-	+0.1	2.375	2.61	25
-do-	-do-	-0.1	2.625	2.39	14
-do-	-do-	-0.3	2.875	2.16	12.5
-do-	-do-	-0.5	3.125	1.94	11

<sup>1/</sup> Equation [9] was used to compute this value.

$$\supseteq d_{10\%} \triangleq d_i + 0.90 (d_f - d_i).$$

#### 4. Frequency response

The response of the simulated mole plow to sinusoidal inputs at the hitch point provides another measure of performance indicating speed-of-response and relative stability. An example frequency response test by computer simulation is shown in Figure 20. It is seen that the resulting oscillations in moling depth are larger in magnitude below the original steady-state depth than above. This occurs because the plow will penetrate to a greater depth faster than it can decrease in operating depth. Since the plow model is nonlinear, sinusoidal inputs of varying amplitude as well as frequency must be used to study and evaluate the system response. A moling depth of 2.5 ft. was used for this analysis since oscillations were larger than if a shallower

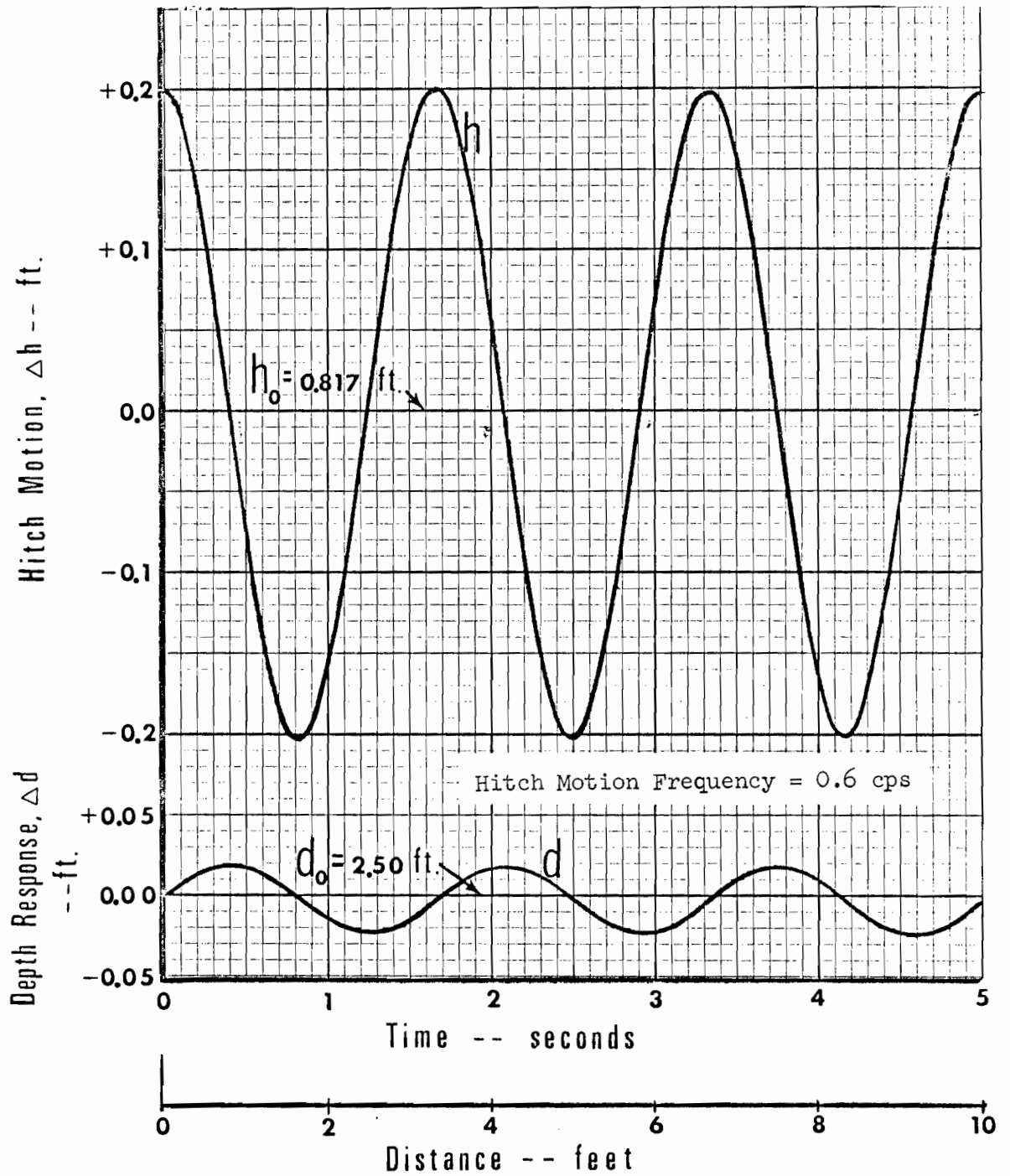


Fig. 20. Example frequency response test of simulated mole plow.

simulated operating depth were used. The plow's frequency response characteristics for maximum hitch oscillations of  $\pm 0.1$  and  $\pm 0.2$  ft. are presented graphically in Figure 21. One can see that for hitch sinusoidal oscillations with a frequency of 1 cycle per second ( $2\pi$  rad./sec.) or greater, the resulting oscillations in the moling depth are very small. This characteristic of response is important when considering automatic depth and grade control systems for the mole plow, which will be covered in detail in the next chapter.

#### 5. Operation to grade

Since the mole plow operates in a slightly nonlinear manner, it is not possible to lay a drain with a given gradient by controlling the hitch point such that it follows a line parallel to the desired drain slope. The results of the computer simulation to illustrate this phenomenon are given in Figure 22. The governing factor in the mathematical model is the  $r$ -term in equation [12] where the hitch height is the only means of input or control. One could control moling depth by regulating the angle between the plow blade and the beam, which in effect changes the  $n$ -term in equation [12], while  $h$  is held constant. Also, simultaneous control of both hitch height and the blade-to-beam angle (that is, both  $h$  and  $n$  in equation [12]), could be implemented to obtain the desired drain gradient. However, these latter two concepts would require nonlinear control of the blade-to-beam angle (for example, by adjusting the large turnbuckle screws on the USDA-ARS mole plow; Appendix A) in order to obtain a linear relationship between hitch height and moling depth. Figure 50 in Appendix B shows the relationship between moling depth and blade-to-beam angle (turnbuckle position)

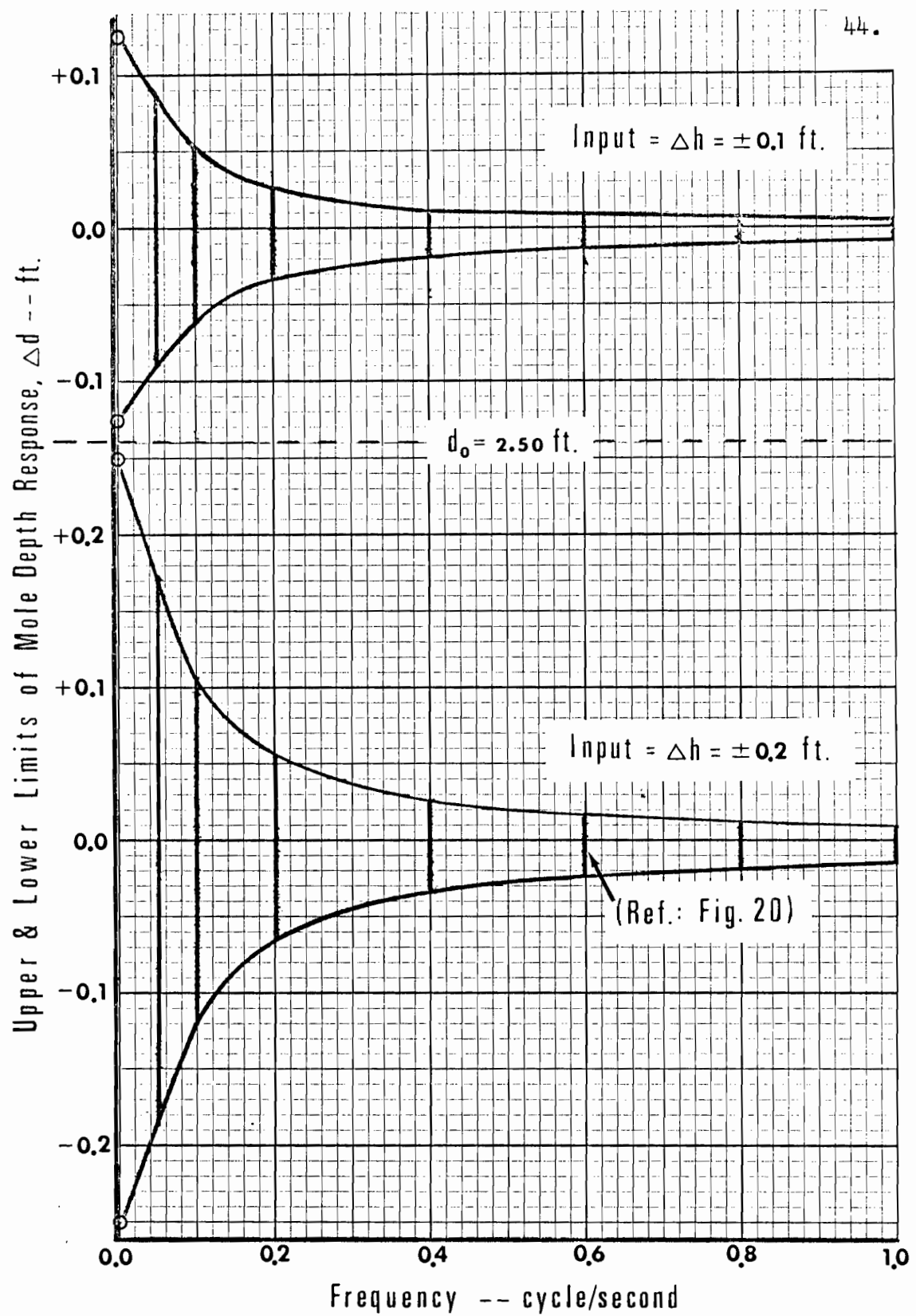


Fig. 21. Frequency response characteristics of simulated plow for two input levels.

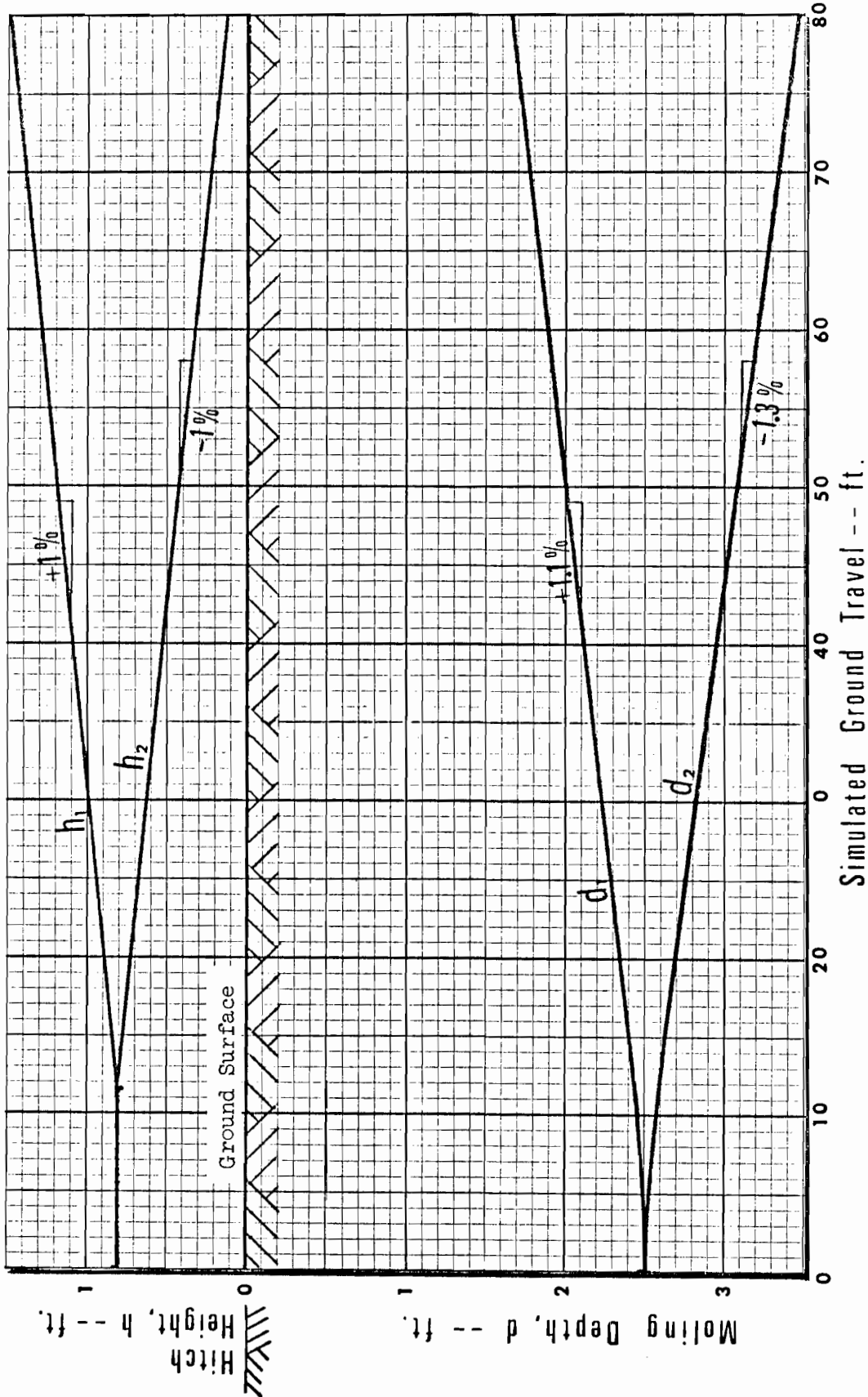


Fig. 22. Simulated mole plow response for level land where the hitch height varies directly proportional to desired uniform drain gradient.

for the USDA-ARS mole plow using field test data. Grade control by controlling only the hitch height was selected, as will be explained in detail in the discussion.

### III. SIMULATION OF LASER GRADE CONTROL SYSTEM

The second objective of this study was to develop a mathematical model of the 'Laserplane" grade control system mounted on a drainage plow (Figs. 4 and 5) for incorporation with the plow model (equation [12]) so that the complete plow and feedback control system could be simulated for study. The purpose of this second simulation study was to develop an analytical method to determine the optimum position for mounting a single laser receiver-controller on the plow frame when regulation of the hitch height is the only means of feedback control for moling depth. The sample results and interpretations presented here are intended only to illustrate the value of this method and not to give any specific design data.

#### A. Simplified Plow Model

To improve the accuracy of the analysis and computer simulation, the input and output variables are defined as perturbation quantities about some steady-state operating level. The hitch height and moling depth may be defined as:

$$h \stackrel{\Delta}{=} h_s + h_p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [20]$$

and

$$d \stackrel{\Delta}{=} d_s + d_p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [21]$$

where  $h_p$  and  $d_p$  are the perturbation variables of interest.

The simulation results of Chapter II reveal that typical hitch accelerations caused by the hydraulic depth adjusting mechanism have an insignificant effect on the moling depth. The plow dynamic model was, therefore, simplified to

$$\ddot{d} + \left( \frac{b^2 c}{J_H} \right) \dot{d} + \left( \frac{b(rd + h - n)}{J_H} \right) R_H = 0 \quad \dots [22]$$

by assuming  $\ddot{h} = 0$ .

Substituting equations [20] and [21] into equation [22], one obtains the following equation describing the mole plow dynamics in terms of perturbation variables:

$$\ddot{d}_p = - \left( \frac{b^2 c}{J_H} \right) \dot{d}_p - \left( \frac{b}{J_H} \right) (r d_p + h_p) R_H \quad \dots [23]$$

## B. Grade Control Components

### 1. Receiver-Controller

The receiver-controller component consists of the laser beam receiving unit, the signal processing circuit, and the output (correction signal) circuit. Time delays, if any, in this component are assumed negligible.

a. Location.--The vertical displacement of a laser receiver mounted on the plow beam a horizontal distance  $x$  behind the hitch can be expressed in terms of hitch height and moling depth. If  $z_s$  is its height above the ground reference plane corresponding to  $h_s$  and  $d_s$ , its perturbation displacement  $z_p$  measured from  $z_s$  would be given by

$$z_p = h_p - \frac{x}{b} (d_p + h_p), \quad \dots [24]$$



for small beam angular movements.

b. Characteristics.--Three types of on-off receiver-controllers were considered in this study plus one proportional receiver-controller for comparative purposes. The operational characteristics of each are given below in terms of a control mode.

(1) "Bang-bang" (on-off): This mode of control is basically one without a dead-zone (true null). Its corrective effort continuously limit-cycles ("hunts") for all stable systems. The frequency and magnitude of "hunt" depends on system parameters such as gain, damping, natural frequency, etc. The input-output relationship of this mode can be expressed mathematically as follows:

$$\text{Output} = \left\{ \begin{array}{l} +\dot{h}_c, z_p < 0 \\ -\dot{h}_c, z_p > 0 \end{array} \right\}, \dots \dots \dots [25]$$

where the output is the voltage signal applied to the hydraulic-actuator, which is expressed as the hydraulic cylinder velocity  $\dot{h}_c$ , and  $z_p$  is the receiver displacement.

(2) Dead-zone, on-off: This mode of control is similar to "bang-bang" except that a dead-zone exists near null. Because of the dead-zone, corrective effort may or may not limit cycle for many systems, depending upon the size of the dead-zone, relative stability of the system, system gain, time lags in the system, etc. Many practical controls in use are of this type. Mathematically, the dead-zone on-off mode of control can be defined as:

$$\text{Output} = \left\{ \begin{array}{l} + \dot{h}_c \quad , \quad z_p < -z_{DZ} \\ 0 \quad , \quad -z_{DZ} \leq z_p \leq +z_{DZ} \\ - \dot{h}_c \quad , \quad z_p > +z_{DZ} \end{array} \right\} \cdot \cdot \cdot \cdot \cdot [26]$$

(3) Digital on-off: This control mode is basically one where the corrective effort occurs in equal and definite steps. It is sometimes called "sampling" control because the polarity of the error is sampled during the dead-time between steps. In effect, it modulates the corrective effort which would occur from either a bang-bang or a dead-zone, on-off receiver. If the receiver unit is of the bang-bang type, this control mode can be expressed mathematically as:

$$\text{Output} = \left\{ \begin{array}{l} 0 \quad , \quad 0 < t < \tau_s \\ + \dot{h}_c \quad , \quad \tau_s < t < \tau_r \quad , \quad \text{and } z_p < 0 \\ - \dot{h}_c \quad , \quad \tau_s < t < \tau_r \quad , \quad \text{and } z_p > 0 \end{array} \right\} , \cdot \cdot \cdot \cdot [27]$$

where  $t$  is reset to zero after each corrective cycle,  $\tau_s$  and  $\tau_r$  are the start and end of the correction time, respectively. It is noted that as  $\tau_s \rightarrow 0$ , the control mode approaches that of the receiver unit (that is, bang-bang in the above example).

The "Laserplane" automatic-grade control system currently in use on tile trenching machines (Fig. 5) employs the digital on-off control mode. The radial rotation of the laser beam at 5 cycles per second to create the "laserplane" effect and permits a corrective signal every 0.200 second. As the laser beam passes the receiver, it triggers a multivibrator circuit to hold the correction signal for the desired duration. On most trenching machines, two durations of corrective signals are set as follows:  $\tau_r - \tau_s = 0.080$  sec. for  $z_p \leq \pm 1$  in., and

$\tau_r - \tau_s = 0.180$  sec. for  $z_p \geq \pm 1$  in. Experience has shown that the shorter correction signal is adequate if hydraulic gain is properly set. It has been determined that the photocell receiver unit for this "Laserplane System" is essentially of the "bang-bang" type.<sup>1/</sup>

(4) Proportional: In this mode of control, corrective effort (e.g., velocity of hitch motion  $\dot{h}_c$  for the plow) is proportional to the magnitude of the error; the proportionality could be either continuous or step-wise. A continuously proportional mode of control can be expressed mathematically as

$$\dot{h}_c = -K_c z_p \dots \dots \dots [28]$$

where,  $K_c \triangleq$  proportionally gain factor. Most proportional controllers have a maximum or "saturation" limit and would be described by

$$\dot{h}_c = \left\{ \begin{array}{l} -K_c z_p \quad , \quad |z_p| \leq z_{pl} \\ -K_c z_{pl} \quad , \quad |z_p| \geq z_{pl} \end{array} \right\} \dots \dots \dots [29]$$

It may be noted that  $(-K_c z_{pl})$  is the maximum value of  $\dot{h}_c$ .

## 2. Hydraulic-Actuator

The hydraulic-actuator component consists of the hitch control linkage, hydraulic cylinders, and actuating valve. A solenoid-type valve was assumed for on-off control modes, and a servo-valve for proportional control. Time lags were assumed associated with the hydraulic valve. Acceleration of the plow hitch by the hydraulic-actuator

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<sup>1/</sup> Personal communication with Mr. T. L. Teach, General Manager, Laserplane Corp., Dayton, Ohio.

was neglected (that is,  $\ddot{h} = 0$ ).

a. Linkage.--The vertical displacement of the USDA-ARS plow hitch by the depth adjusting linkage is essentially directly proportional to the motion of the active hydraulic cylinder (Appendix A). The linkage system is also kinematically designed such that the relative horizontal motion between the mole M and the tractor is less than 0.1 ft. for reasonably large hitch displacements (e.g.,  $\Delta h = \pm 0.75$  ft.), from a centered hitch position ( $h = 1.17$  ft.).

b. Characteristics.--For the on-off control modes, the delayed reaction of the hydraulic solenoid valve is modeled as a pure time lag  $\tau_L$ , and the hydraulic cylinder motion velocity  $\dot{h}_c$  is modeled as a step function; the upwards and downwards velocities were assumed equal in magnitude. The input-output relationships of the modeled solenoid valve-cylinder subsystem is shown in Figure 23; the input function  $\dot{h}_c$  is provided by the output of the modeled receiver-controller. This model closely describes the operational characteristics of the hydraulic valve-cylinder system on the USDA-ARS mole plow as shown in Figure 47 (Appendix A), where  $\tau_L = 0.15$  sec.

For the proportional control mode (which is included here for comparative purposes), the hydraulic servo-valve is assumed to have a first-order time lag characteristic  $\tau_L$ , and the cylinder motion is assumed directly proportional to the modeled servo-valve output, expressed as  $\dot{h}_c(t - \tau_L)$ .

### C. System Block Diagram

The block diagram of the mole plow and laser beam (or plane) feedback control system is shown in Figure 24. The mathematical model

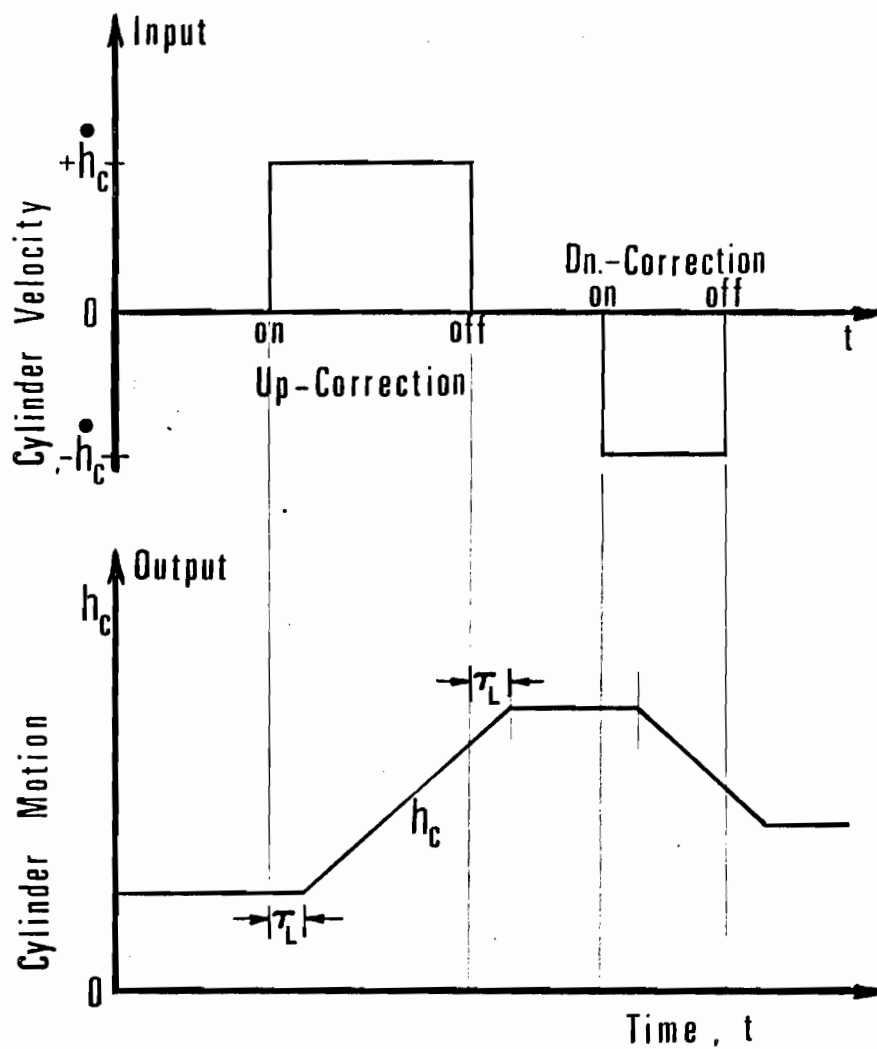


Fig. 23. Input-output characteristics of the modeled solenoid valve-cylinder subsystem.

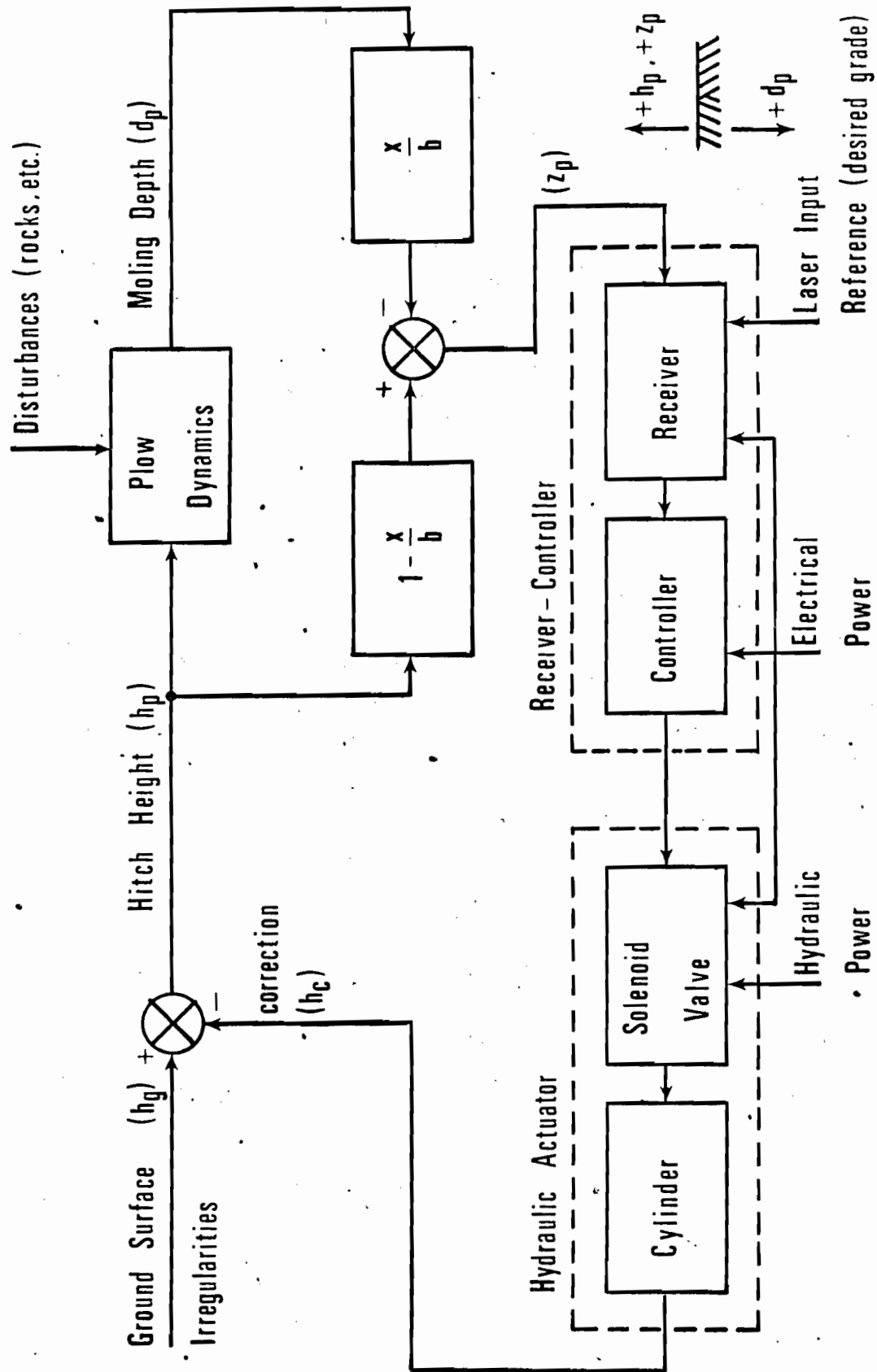


Fig. 24. Block diagram of the mole plow and laser depth-and-grade feedback control system.

for the 'plow dynamics' block is equation [23], which is nonlinear and could not be represented in conventional transfer function form. One might use the "describing function" approximation to analyze the plow stability if only sinusoidal hitch motions were considered,<sup>1/</sup> but little knowledge would be gained about settling time, transient-response, etc. Thus, computer simulation seems the best analysis approach.

#### D. System Performance

The object of automatic grade control of the mole plow is to regulate the moling depth such that the formed drain channel lies at a specific depth and gradient, independent of ground surface irregularities. The accuracy with which the channel can be maintained at the required depth during forward travel is the performance criterion. In control system design, one generally selects the simplest control mode which will provide the required performance. Based on the mole plow frequency response given in Chapter II, and past experience, it was projected that some type of on-off control which limit cycles ("hunts") at a frequency of about 1 cycle/sec. would be satisfactory.

##### 1. Performance Requirements

For purposes of this study, the minimum requirements for acceptable grade control are defined as follows:

- (1) The maximum reverse grade permitted in any section of a drain channel is a 0.05 ft. elevation rise in the channel towards the outlet.

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<sup>1/</sup> Doebelin (1962, pp. 207-214).

(2) Localized departures of the drain channel from the desired grade-line should not exceed  $\pm 0.05$  ft. for gradients up to 1 percent. For gradients steeper than 1 percent, greater departure may be permissible, but in control system design the performance at the small gradient is the most critical.

The grade control requirements stated above exceed those suggested by several investigators, such as by Schwab (1951), Ede (1965), and Hermsmeier and Willardson (1970).

2. Performance Evaluation

Evaluation of a given computer simulation run in accordance with the minimum grade control requirements stated in Section 1 above can be accomplished by visual inspection. However, the use of an "overall performance criteria" is also helpful in evaluating the difference between various control systems which meet the minimum requirements. Most such performance evaluating criteria involve integrating some function of the error term,  $\epsilon$ , the difference between moled and desired depths, over a specified time interval T (or distance of ground travel  $vT$ ). The control system giving the smallest integral is called the "optimum system". Two performance criteria used for evaluating control accuracy are:<sup>1/</sup>

(1) The error-integral:

$$\int_0^{vT} \epsilon d(vt) \dots \dots \dots [30]$$

If this integral is zero, the average mole depth is at the desired depth.

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<sup>1/</sup> Doebelin (1962, pp. 161-162).



(2) The absolute-error-integral:

$$\int_0^{vT} |\epsilon| d(vt) \dots \dots \dots [31]$$

This criterion discriminates against sluggish overdamped, and fast excessively underdamped systems.

E. Simulation

To simulate the USDA-ARS mole plow with a "Laserplane" automatic grade control system, two additional simplifying assumptions were made:

(1) The laserplane was assumed to provide a continuous reference rather than an impulse every 0.200 second. Thus, in the simulated system, the polarity of the error could change at any time rather than only once every laser beam revolution. However, for any 0.200 second cycle and at a ground speed of 2 ft./sec., this introduces a ground travel error of  $\pm 0.2$  ft. Since this error would occur in a random order, it was assumed that on the statistical average, the simulated system performance would closely approximate that of the actual system.

(2) It was assumed that ramp, saw-tooth, and sine-wave inputs, or their combination, would provide evaluation of the simulated laser grade control system on the mole plow. No attempt was made to use random inputs; this was left for future study. Sample simulation results are given here to show the method of analysis and interpretative procedures.

1. Analog Computer Circuits

The magnitude-scaled computer circuit used to simulate the dynamic response of the USDA-ARS mole plow with the simplified Laserplane automatic

grade control is shown in Figure 25.<sup>1/</sup> This program solves the perturbation equations [23] and [24] for the parameter constants of the plow given in Table 1 and for  $d_o = 2.5$  ft., and  $\tau_L = 0.15$  sec. The "bang-bang" type of receiver-controller is shown in Figure 25. Analog computer circuits for the dead-zone on-off type, and the digital on-off type are given in Figures 26 and 27, respectively. The computer circuit for the proportional-type receiver-controller with a limiter, and the accompanying hydraulic-actuator<sup>2/</sup> is given in Figure 28.

## 2. "Bang-Bang" Mode of Control

Several simulation runs were made to illustrate typical input-output relations for various system parameters.

a. Limit cycling.--A "bang-bang" type receiver-controller in the laser control system will cause a limit-cycle in the hitch motion  $h_p$ , even on a uniform ground surface ( $h_g \equiv 0$ , desired gradient 0), for all receiver positions on the plow beam (i.e.,  $0 \leq x \leq 11.5$  ft.), as shown in Figure 29, where the maximum hydraulic cylinder speed  $\dot{h}_c = 0.20$  ft./sec. was used. It is seen that the amplitude and frequency of the hitch motion limit-cycle is about +0.020 ft. to -0.015 ft. at 1.5 cycle/sec. for  $0 \leq x \leq 8$  ft. At  $x = 10$  ft., the limit-cycle nearly doubles in magnitude and frequency is lower, thus causing more oscillation in the

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<sup>1/</sup> The hydraulic-actuator circuit used was developed in earlier work reported by Fouss, J. L. and M. Y. Hamdy (1970), "Simulation of a laser beam automatic depth control". ASAE Paper No. 70-531.

<sup>2/</sup> The first-order time lag in the hydraulic-actuator is simulated with a circuit for the first-order Pade' Approximation; Reference: "Handbook of analog computation", Second Edition, Electronic Associates, Inc., Princeton, New Jersey, 1965, p. 227.

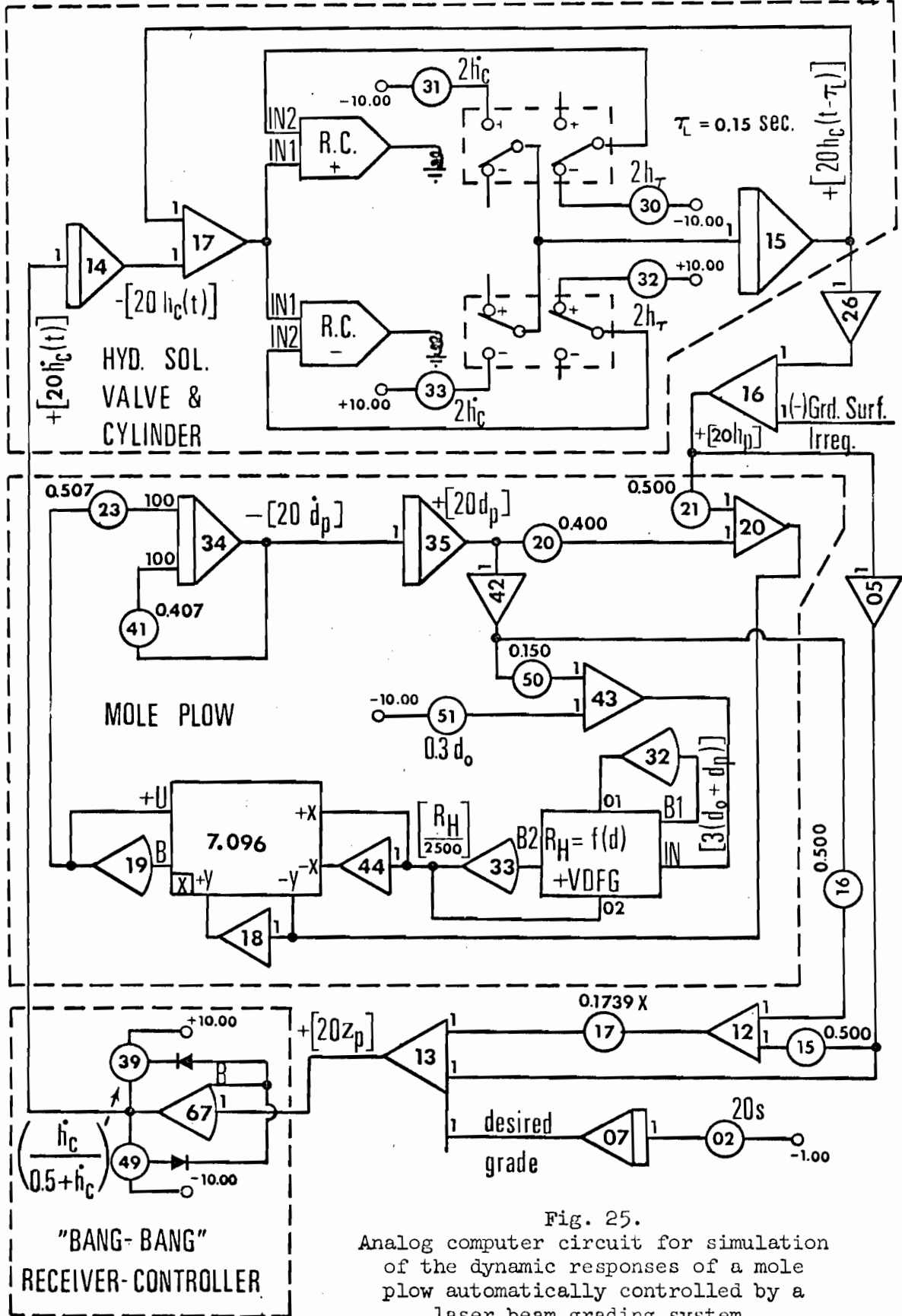


Fig. 25.  
Analog computer circuit for simulation of the dynamic responses of a mole plow automatically controlled by a laser beam grading system.

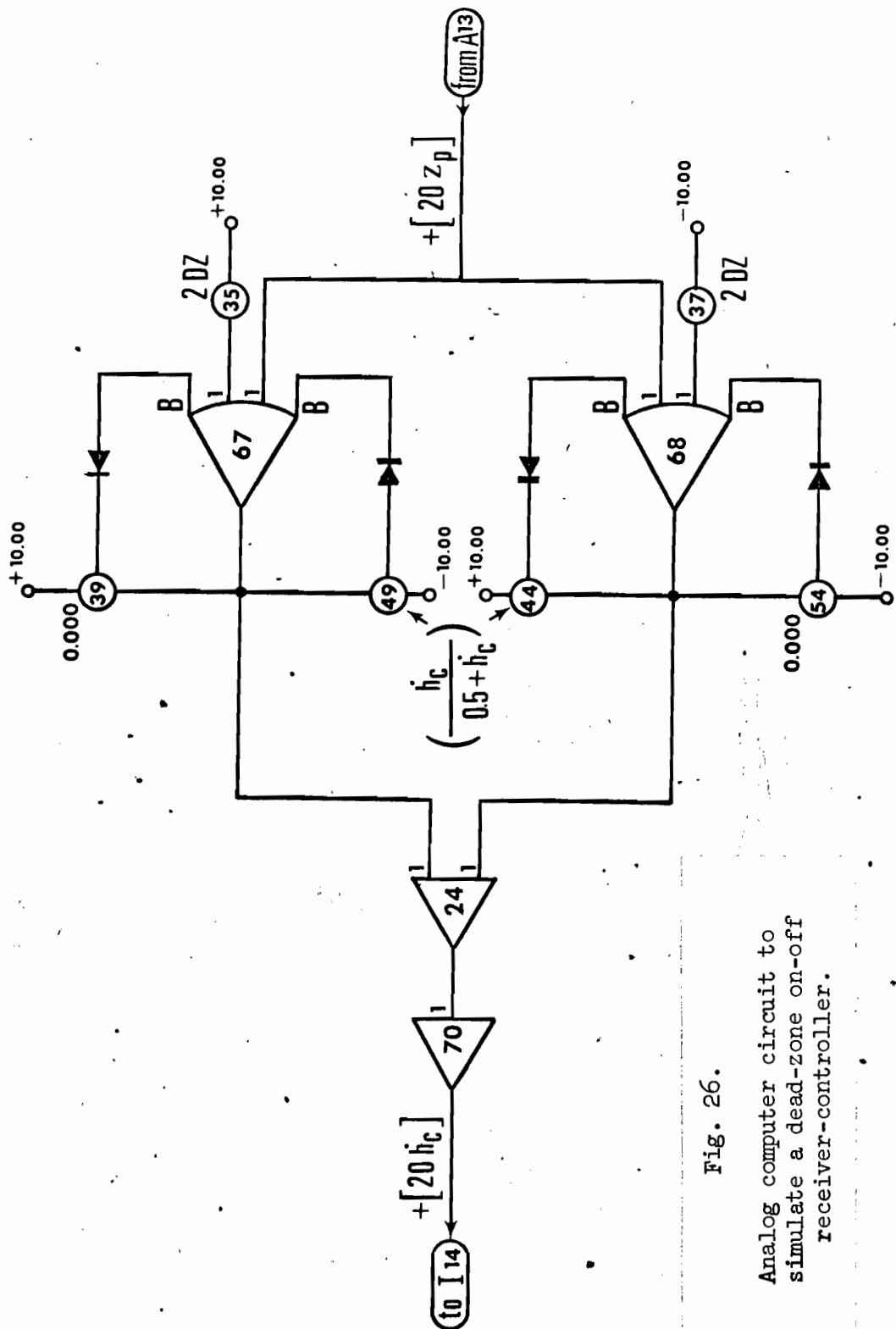


Fig. 26.

Analog computer circuit to simulate a dead-zone on-off receiver-controller.

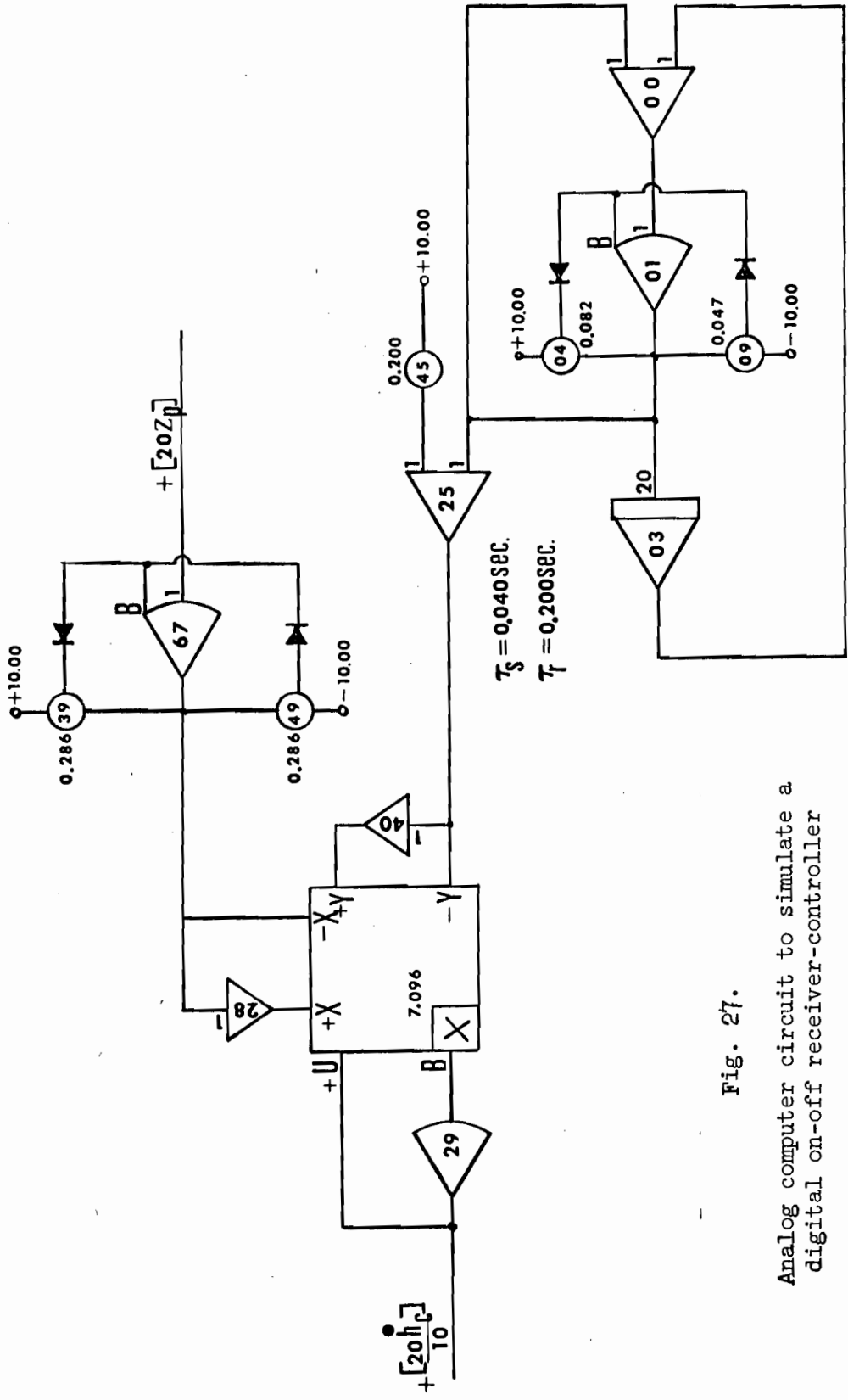


Fig. 27.  
Analog computer circuit to simulate a digital on-off receiver-controller

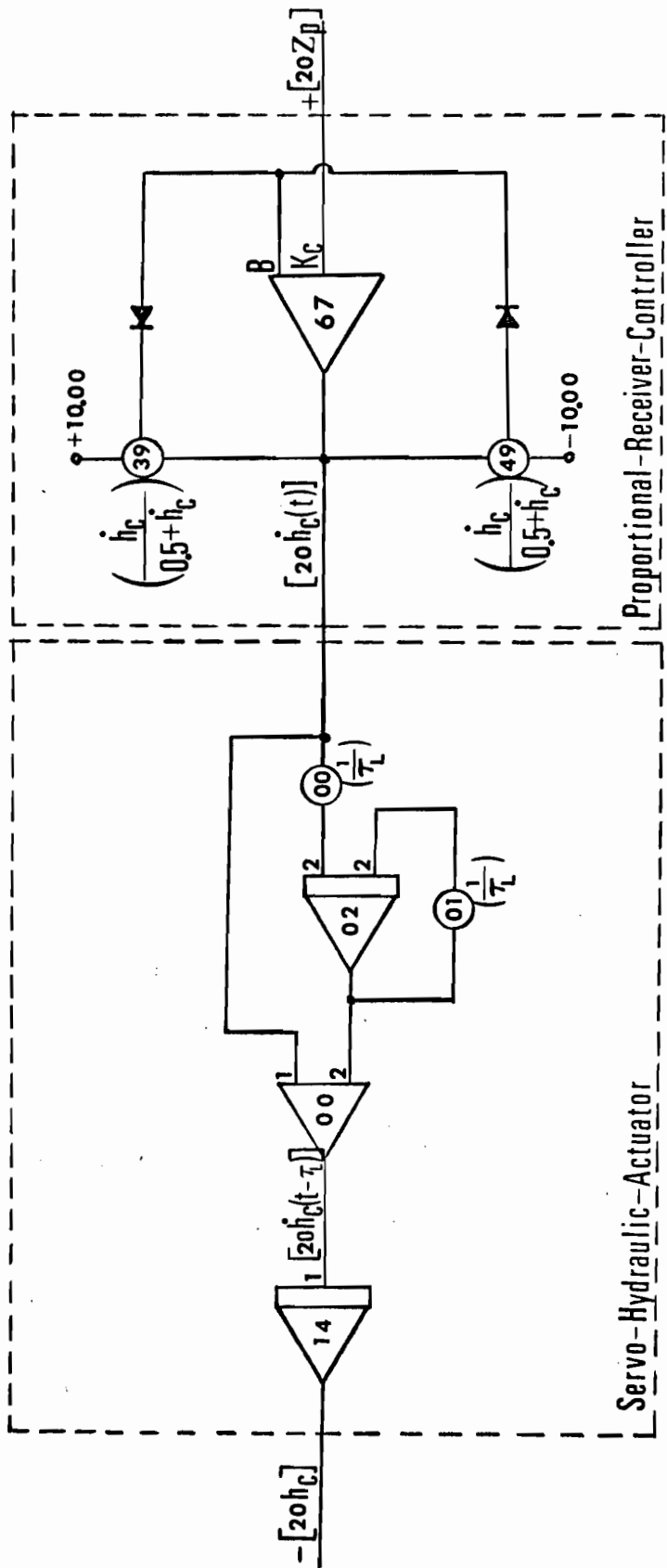


Fig. 28. Analog computer circuit to simulate a proportional receiver-controller with a corrective effort limiter, and a servo-hydraulic-actuator with a first-order time lag.

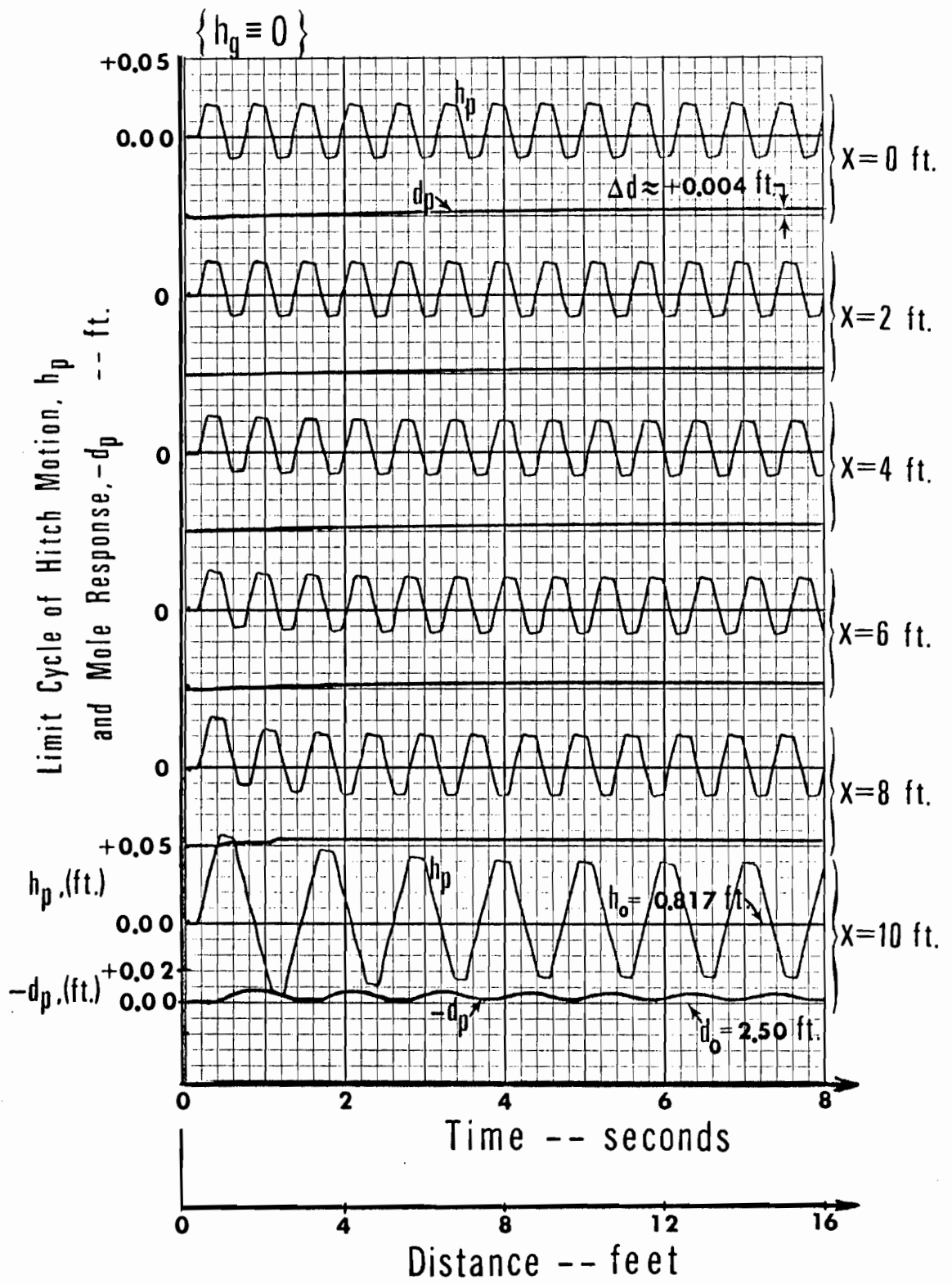


Fig. 29. Natural limit-cycle of "bang-bang" receiver-controller for several receiver locations and a uniform ground surface.

moling depth. The accuracy of control, however, meets the minimum requirements. Changing the hydraulic cylinder speed  $h_c$  would alter both the magnitude and frequency of the limit-cycle.

For all receiver positions  $x \leq 10$  ft., it is noted that the moling depth does not remain at the steady-state depth  $d_o$ , but stabilizes (with only slight oscillation) at a position about 0.004 ft. above the desired depth. However, it is also noted that the average dynamic hitch position is about 0.003 ft. above the normal steady-state height (Figure 29). Using the effective steady-state gain factor ( $\frac{1}{r} = 1.25$ ), the shift in the moling depth would be estimated as  $(0.003)(1.25) \approx 0.004$  ft., which is the same as the simulated dynamic shift. The shift from the normal steady-state level probably results because up and down response differ.

b. Receiver location for sine-wave input.--A sine-wave with an amplitude of  $\pm 0.1$  ft. and frequency of about 0.087 cycle/sec. was selected for the simulated ground surface  $h_g$  (at a ground speed of 2 ft./sec., this is one-half a cycle for one plow beam length of travel, that is, 11.5 ft.). The simulation results for three receiver locations,  $x = 0$ , 8, and 11.5 ft., are given in Figures 30, 31, and 32, respectively. The desired gradient was zero. For the receiver located over the plow blade ( $x = 11.5$  ft.), the system relative stability was poor, and the control accuracy did not meet the minimum requirements. The value of the error-integral after 50 ft. of simulated travel indicates that the receiver location of  $x = 8$  ft. is better than for  $x = 0$  (i.e., at the hitch). Other simulation results (not shown) revealed that for  $x = 4$  ft., performance was essentially the same as for  $x = 8$  ft. However, for the nonlinear plow-control system, the selection of optimum receiver location cannot be



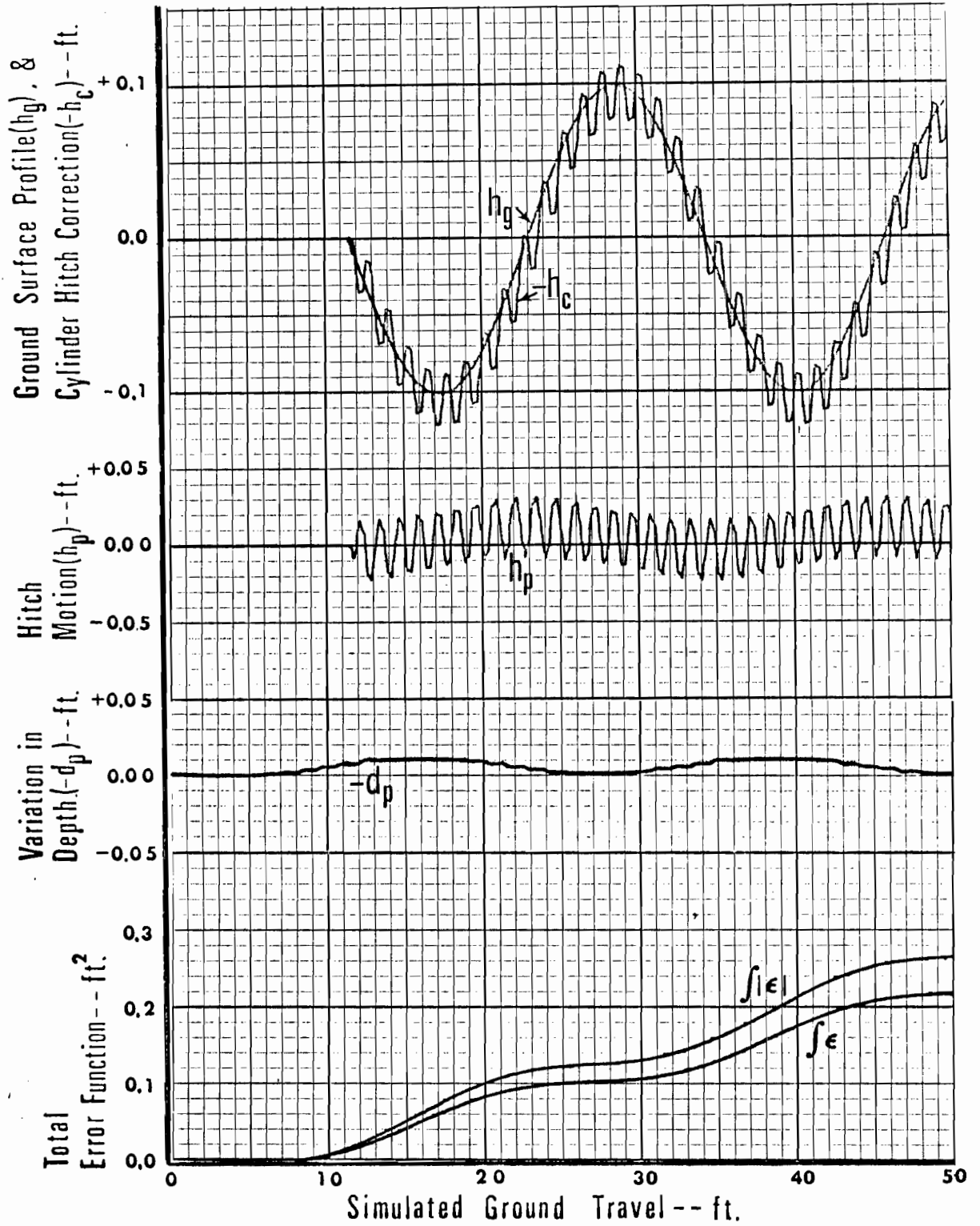


Fig. 30. "Bang-bang" mode of control with receiver at hitch (  $x = 0$  ft.) for sine-wave ground surface.

X = 8 ft.

66.

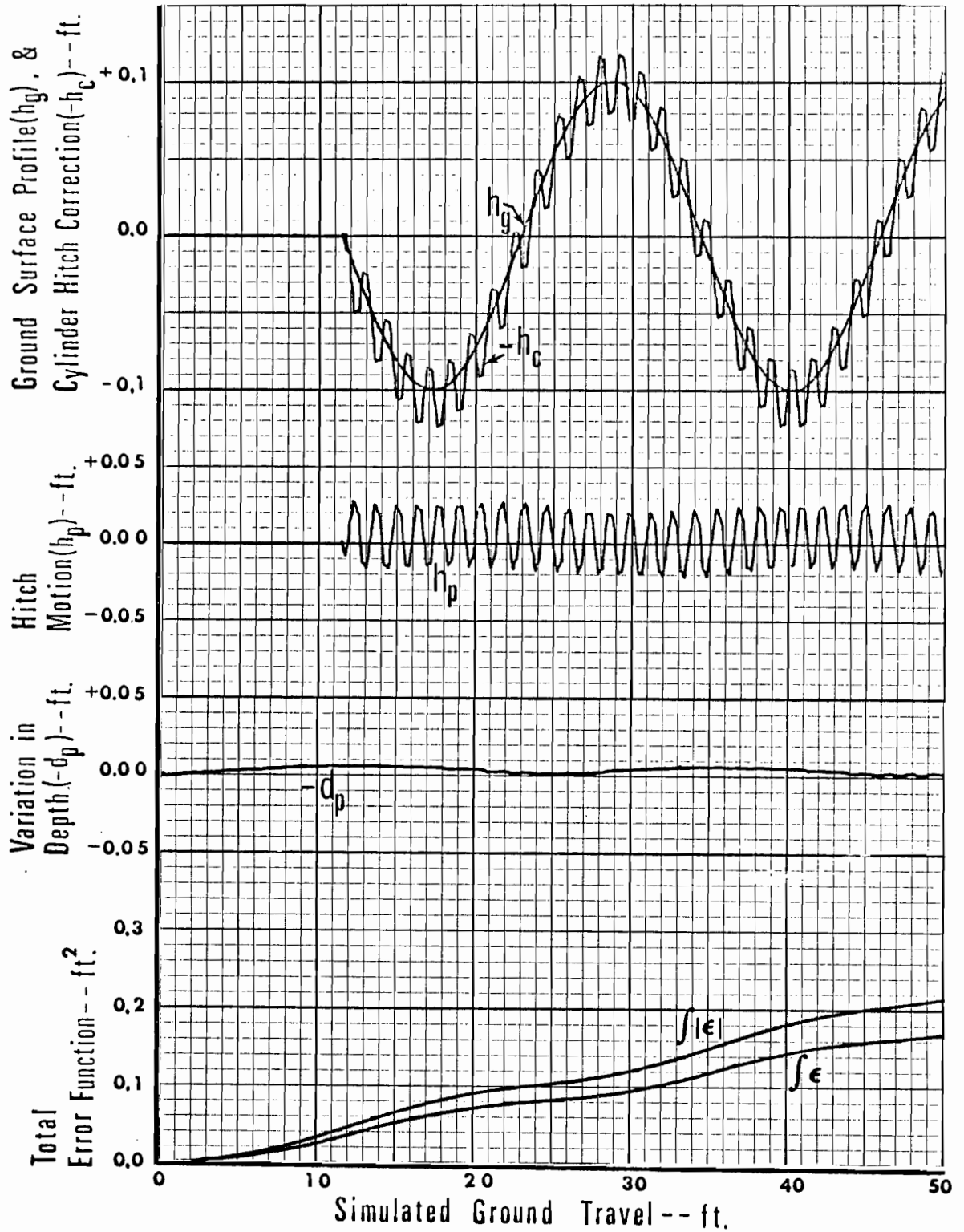


Fig. 31. "Bang-bang" mode of control with receiver 8 ft. back of hitch ( $x = 8$  ft.) for sine-wave ground surface.

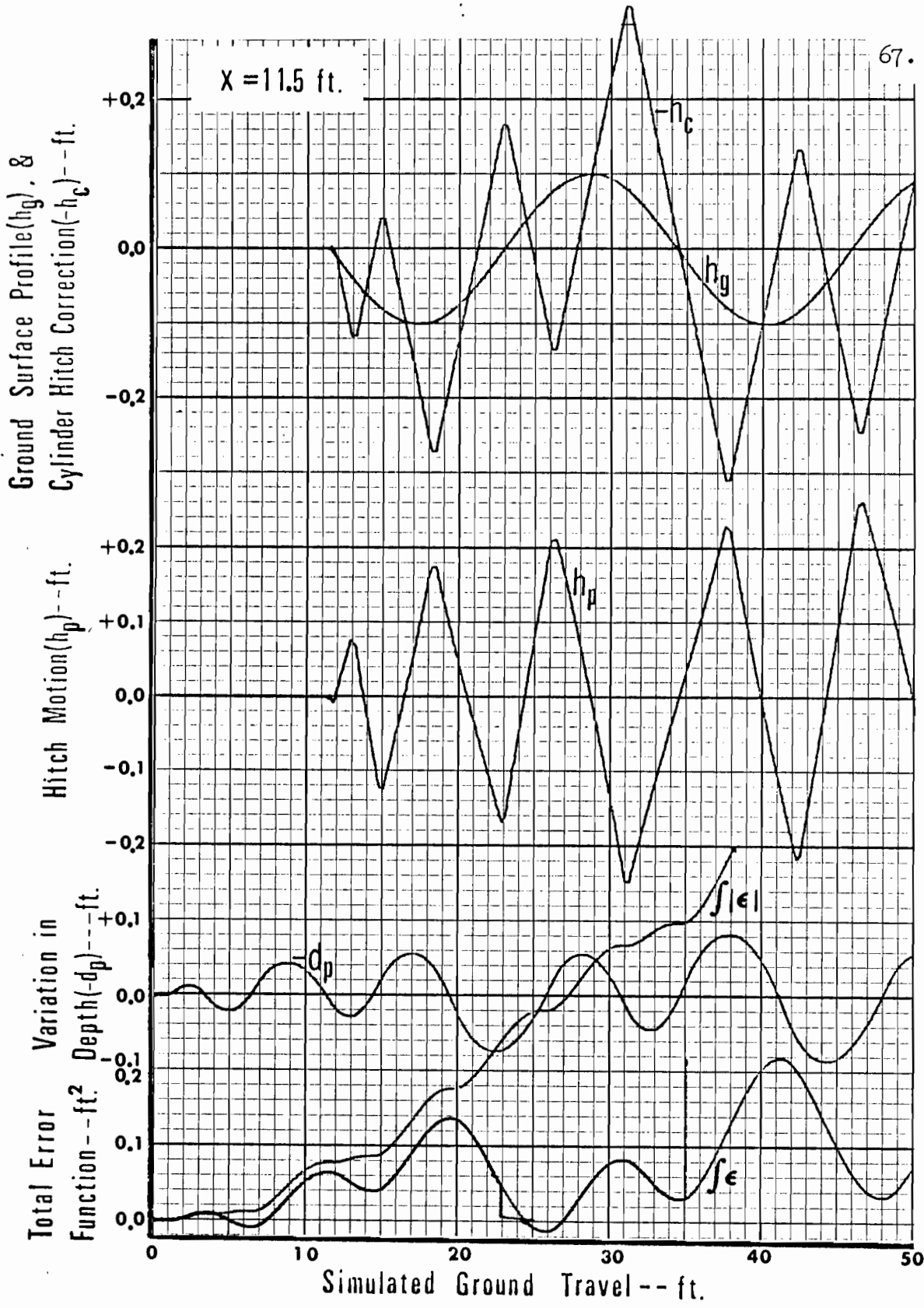


Fig. 32. "Bang-bang" mode of control with receiver over blade (  $x = 11.5$  ft.) and a sine-wave ground surface.

made on the basis of one particular standardized input.

c. Effect of input amplitude and frequency.--To compare the performance for simulated sine-wave ground surface inputs of varying frequency and amplitude, three inputs  $h_g$  were selected as follows:  $\pm 0.1$  ft. at 0.087 cps;  $\pm 0.1$  ft. at 0.050 cps; and  $\pm 0.2$  ft. at 0.087 cps. Simulation results are given in Figures 33, 34, and 35, respectively, for a constant receiver location of  $x = 5.75$  ft., which is the center of the plow beam. The value of the error-integral after 50 ft. of simulated travel indicates that grade control accuracy is nearly equal for all three inputs. The results in Figure 35 indicate, however, if the input amplitude had been larger than  $\pm 0.2$  ft., a hydraulic cylinder speed  $\dot{h}_c > 0.20$  ft./sec. would have been needed to avoid a loss in performance.

d. Effect of changes in ground slope.--To evaluate the effect of changes in the normal land slope along the path where a drain is installed, a saw-toothed simulated ground surface input was used where the slope change was from +1 percent to -1 percent. The desired drain gradient was zero. Simulation results are given in Figure 36. Note the shift in the dynamic steady-state error in moling depth after the change in slope of the ground surface. The error-integral at 50 ft. indicates grading accuracy comparable with the large sine-wave input shown in Figure 35.

e. Operation-to-grade on flat land.--To evaluate the ability for the automatically controlled mole plow to install a drain to specified grade, where the average land slope is zero, several simulation runs were made for varying receiver locations. The simulated laser input reference was set to uniformly vary by 0.5 ft. per 100 ft. of simulated travel;

$X = 5.75 \text{ ft.}$

69.

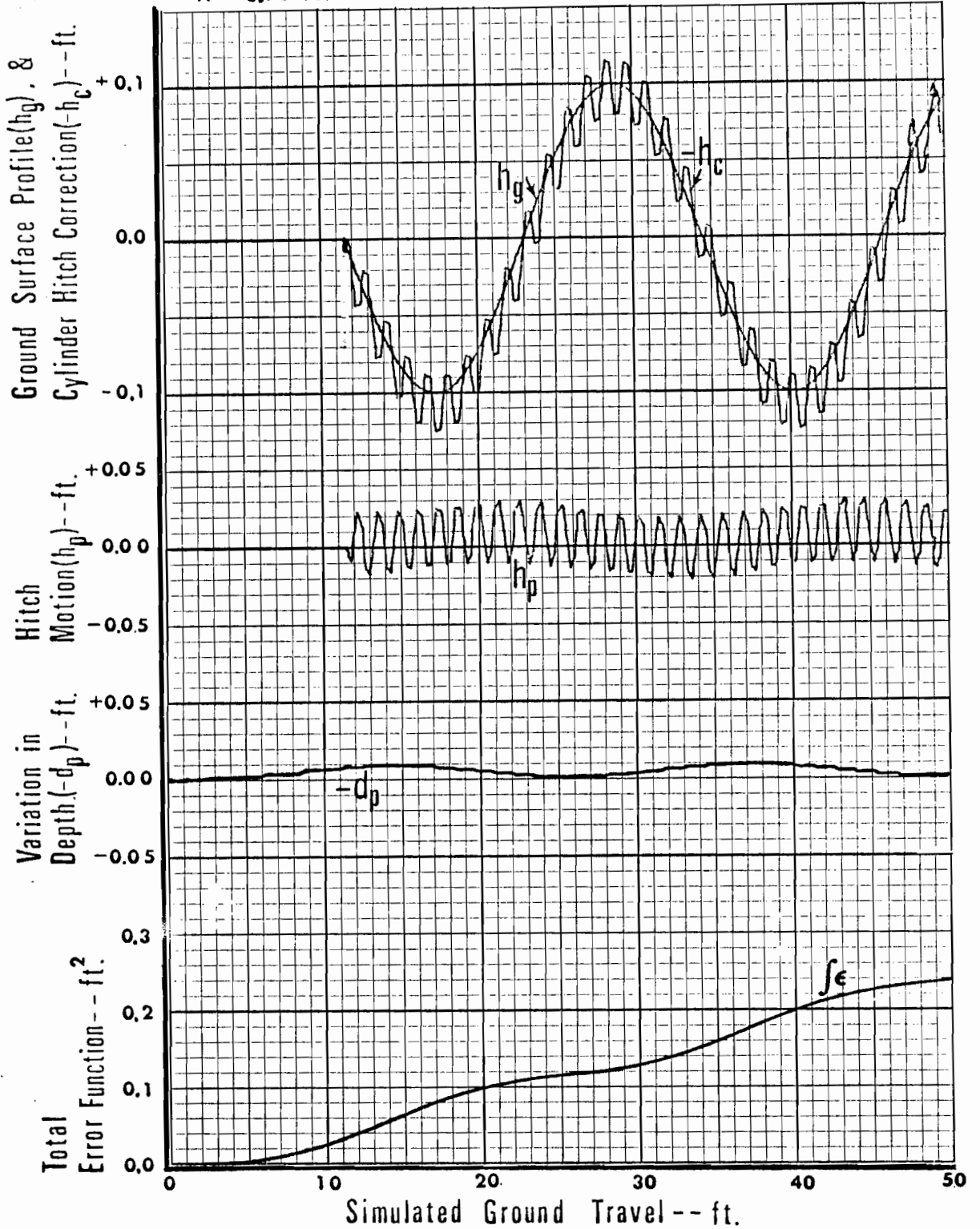


Fig. 33. "Bang-bang" mode of control grading accuracy in relation to amplitude and frequency of simulated sine-wave ground surface;  $\pm 0.1 \text{ ft.}$  at  $0.087 \text{ cycle/second.}$

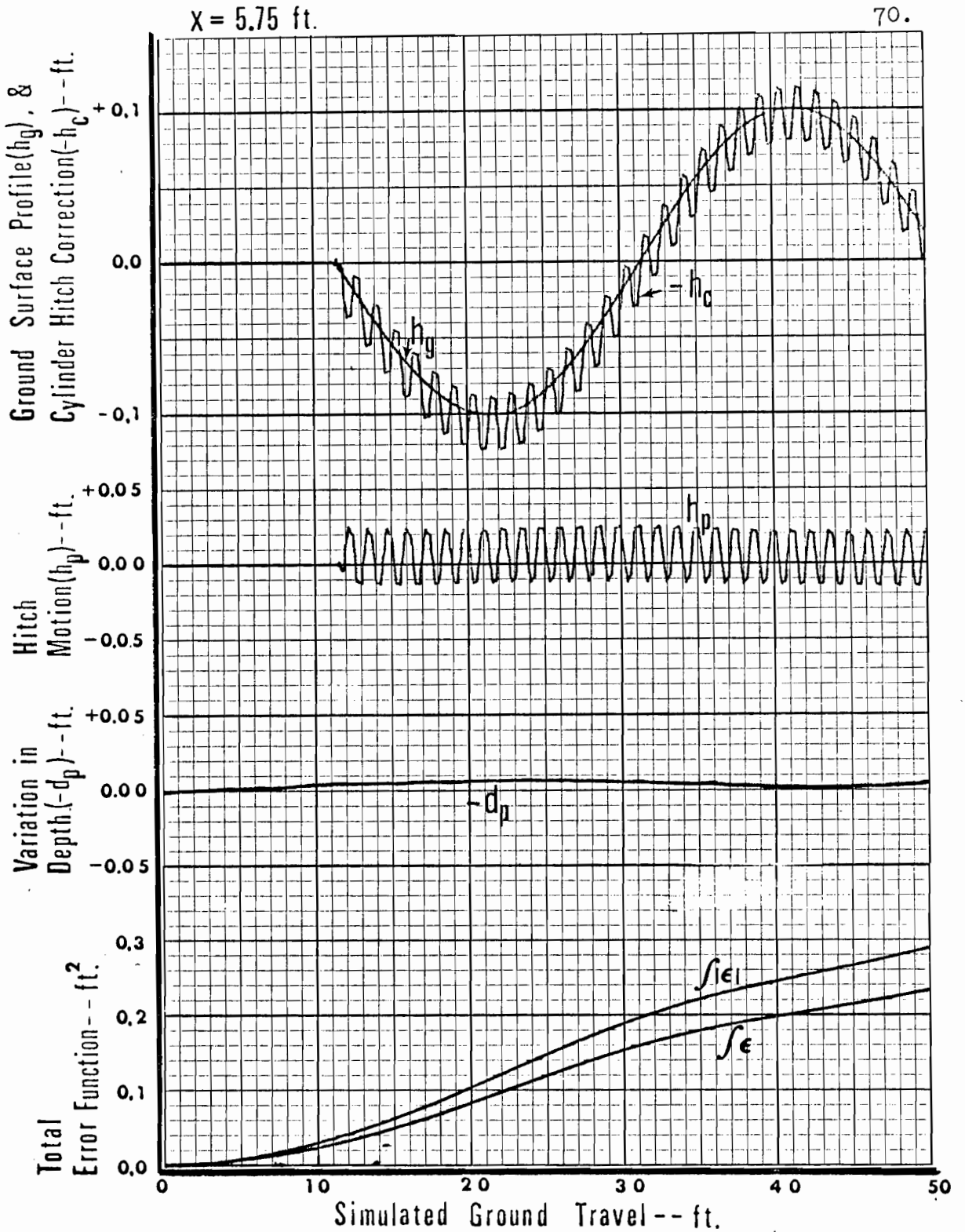


Fig. 34. "Bang-bang" mode of control grading accuracy in relation to amplitude and frequency of simulated sine-wave ground surface;  $\pm 0.1$  ft. at 0.050 cycle/second.

$x = 5.75$  ft.

71.

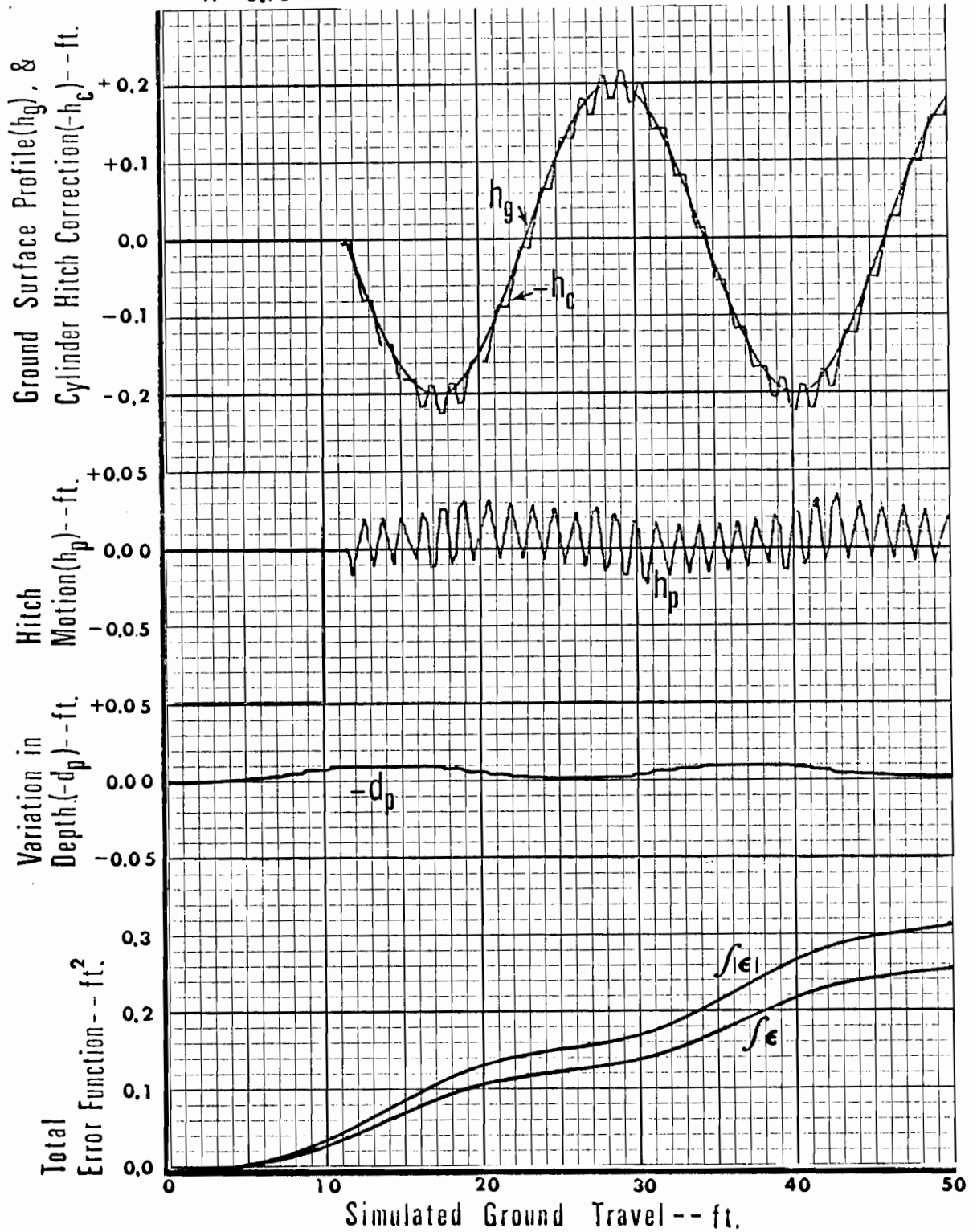


Fig. 35. "Bang-bang" mode of control grading accuracy in relation to amplitude and frequency of simulated sine-wave ground surface;  $\pm 0.2$  ft. at 0.087 cycle/second.

$X = 5.75 \text{ ft.}$

72.

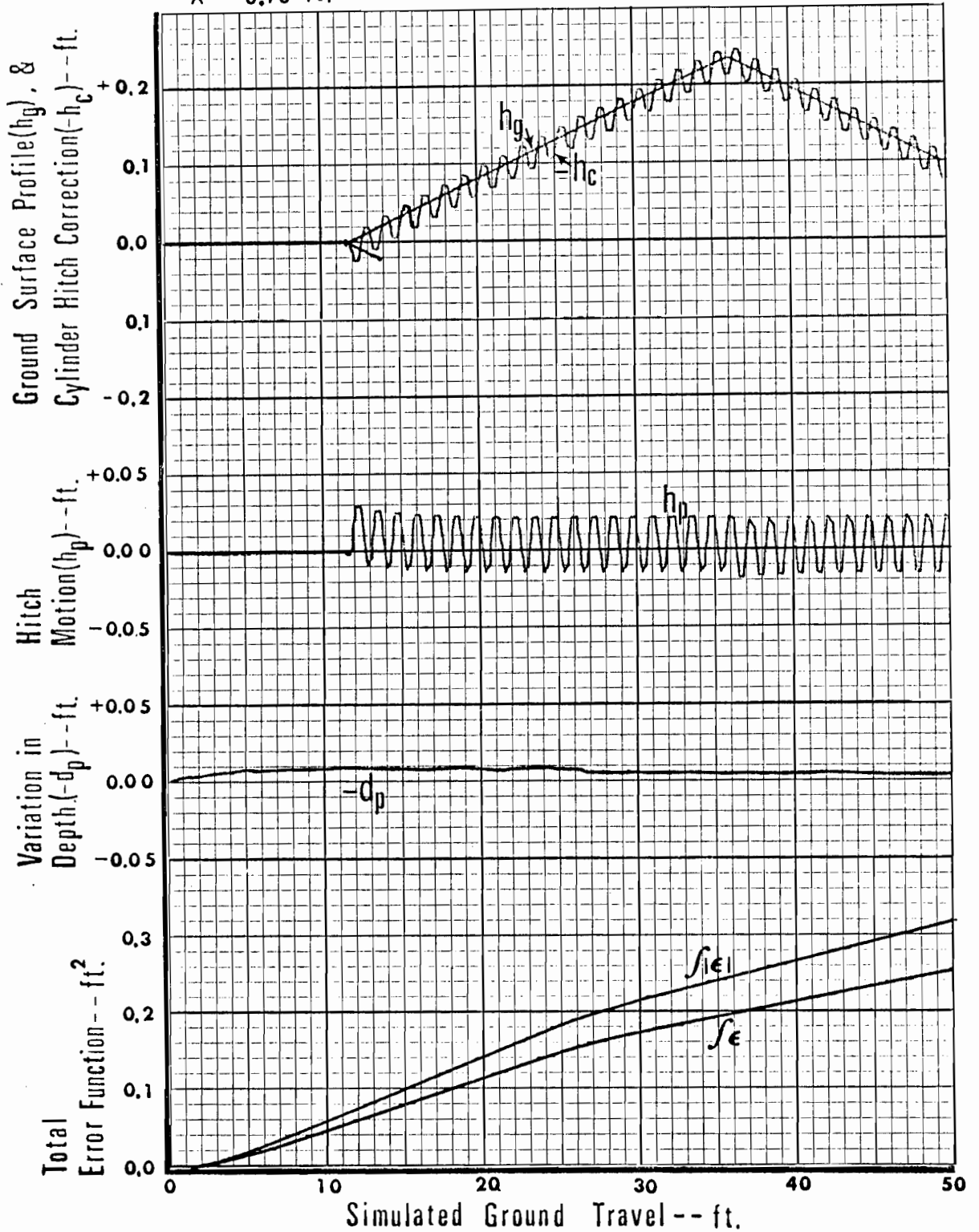


Fig. 36. Effect of ground slope changes on grade control accuracy with "bang-bang" control mode.



that is, at 0.5 percent slope. The simulation results for a horizontal ground surface and  $x = 4, 7,$  and  $10$  ft., are given in Figure 37. It can be seen that a receiver location of  $x = 10$  ft. maintained the channel closest to the set 0.5 percent grade-line; for locations  $x \leq 7$  ft. a gradual drift above a 0.5 percent grade-line occurred; and if the ground travel had been simulated for 100 ft., the departure would probably have exceeded the 0.05 ft. permissible. In subsequent computer runs for  $x = 7$  and  $10$  ft., a saw-tooth simulated ground surface profile was imposed (average slope remained at zero), and again the receiver location  $x = 10$  ft. provided better grade control (Figure 38).

### 3. Dead-Zone On-Off Control Mode

If the receiver-controller has a dead-zone, a "natural" limit-cycle may or may not occur for all receiver locations, depending upon the magnitude of the dead-zone, DZ. In general, the tendency to limit-cycle increases as  $x \rightarrow 0$ . To compare the grade control performance of the "bang-bang", and dead-zone on-off control modes, the simulation runs shown in Figures 39, 40, and 41 were made for comparison with Figures 37 and 38. The dead-zone simulated was about  $\pm 0.011$  ft. and  $\dot{h}_c$  remained at  $\pm 0.20$  ft./sec. At a given receiver location, the major difference is that the moling depth fluctuates more for the dead-zone receiver-controller; compare Figures 38 and 41 (for  $x = 7$  ft.). Moving the dead-zone receiver unit forward on the plow beam would tend to reduce the fluctuations in moling depth, but the average departure from the desired grade-line would increase. This kind of "trade-off" is common in most automatically controlled systems.

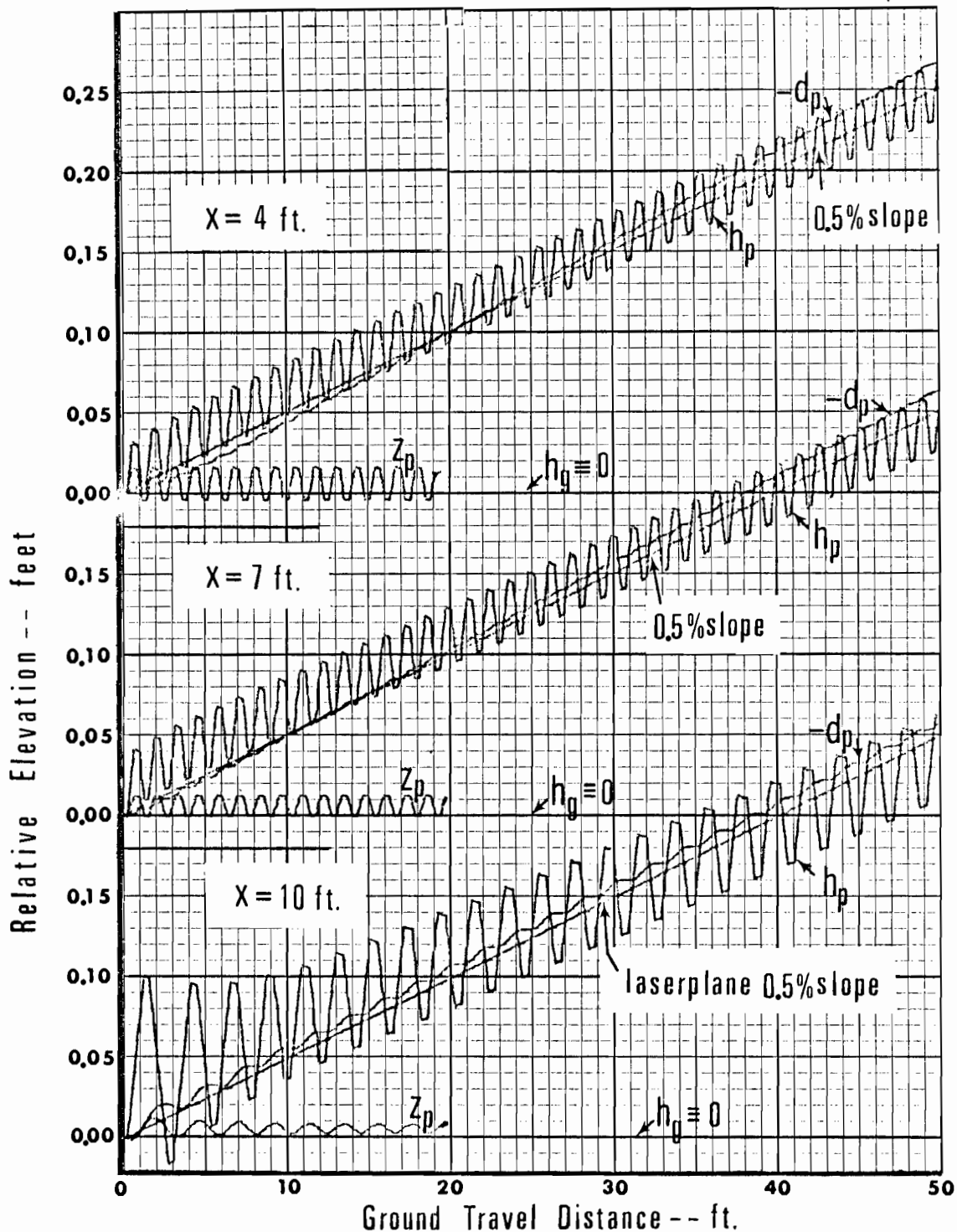


Fig. 37. Effect of laser receiver location on grade of mole channel (bang-bang control mode) where the laser-plane reference is set at 0.5 percent slope, and the ground surface is horizontal.

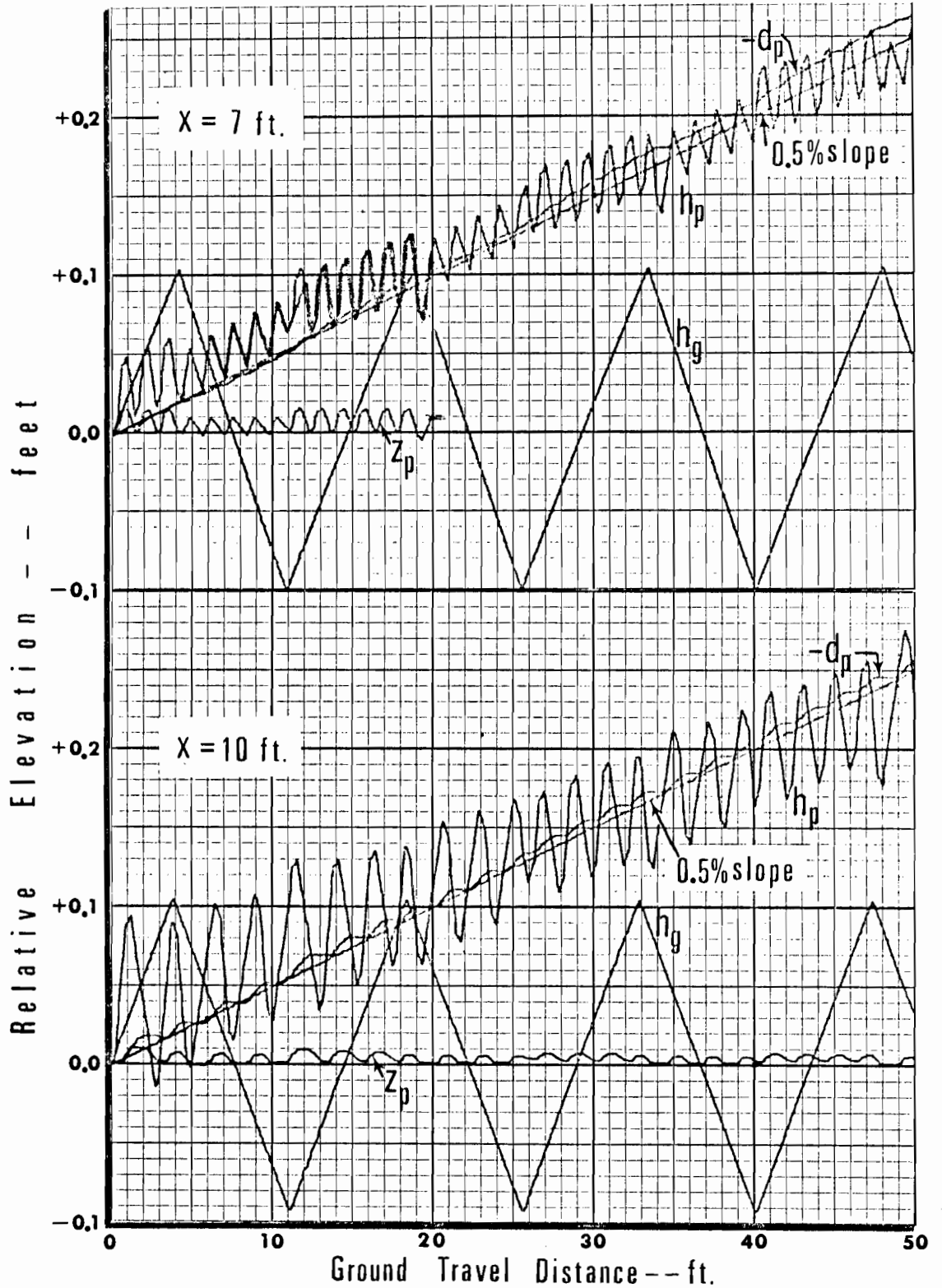


Fig. 38. Grading accuracy of "bang-bang" control mode where the specified grade is 0.5 percent and simulated ground surface is a sawtooth (average land slope is zero).

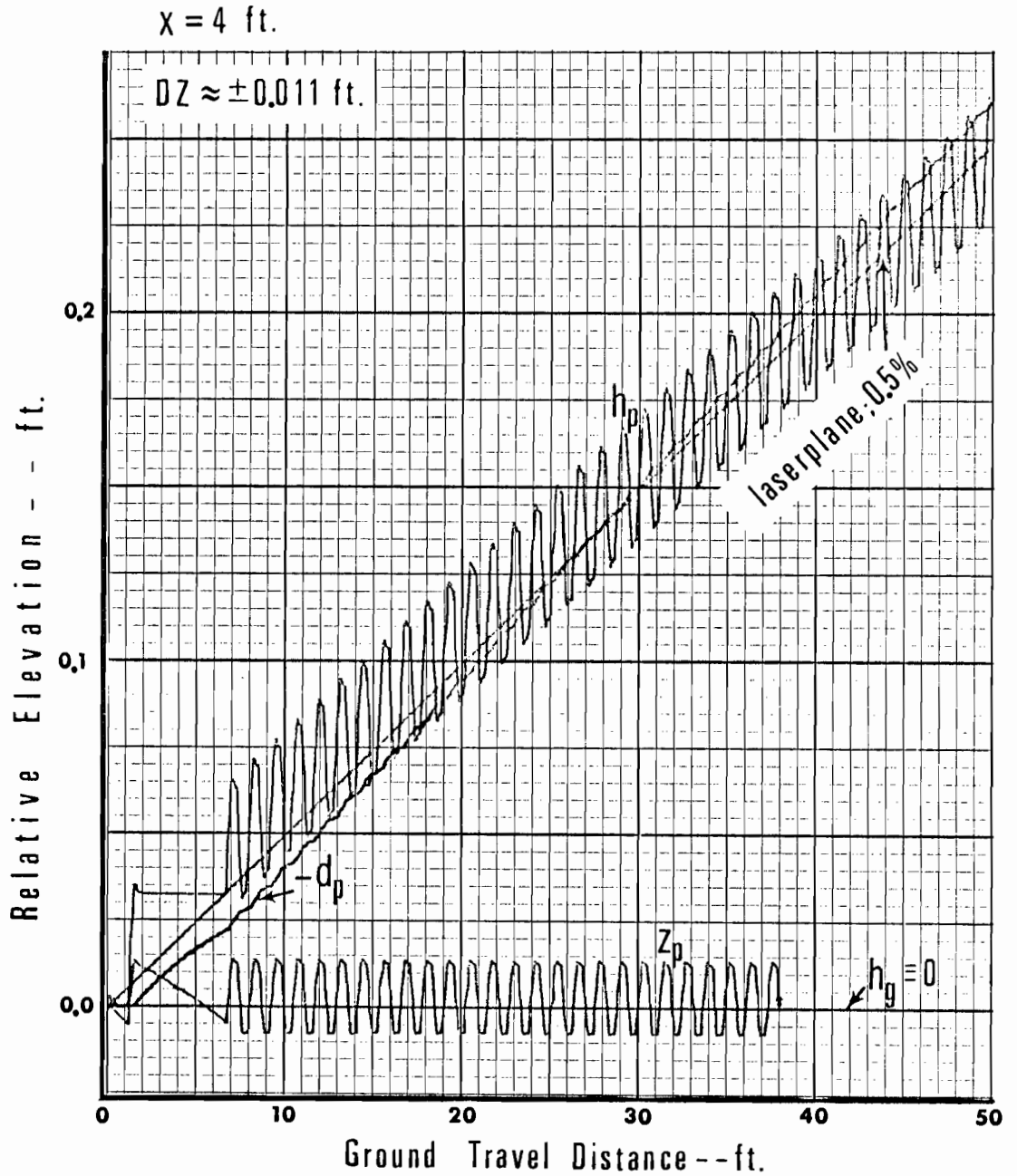


Fig. 39. Dead-zone on-off control mode grading accuracy for level land and 0.5 percent specified gradient;  $DZ \approx \pm 0.011$  ft.;  $x = 4$  ft.

$x = 7$  ft.

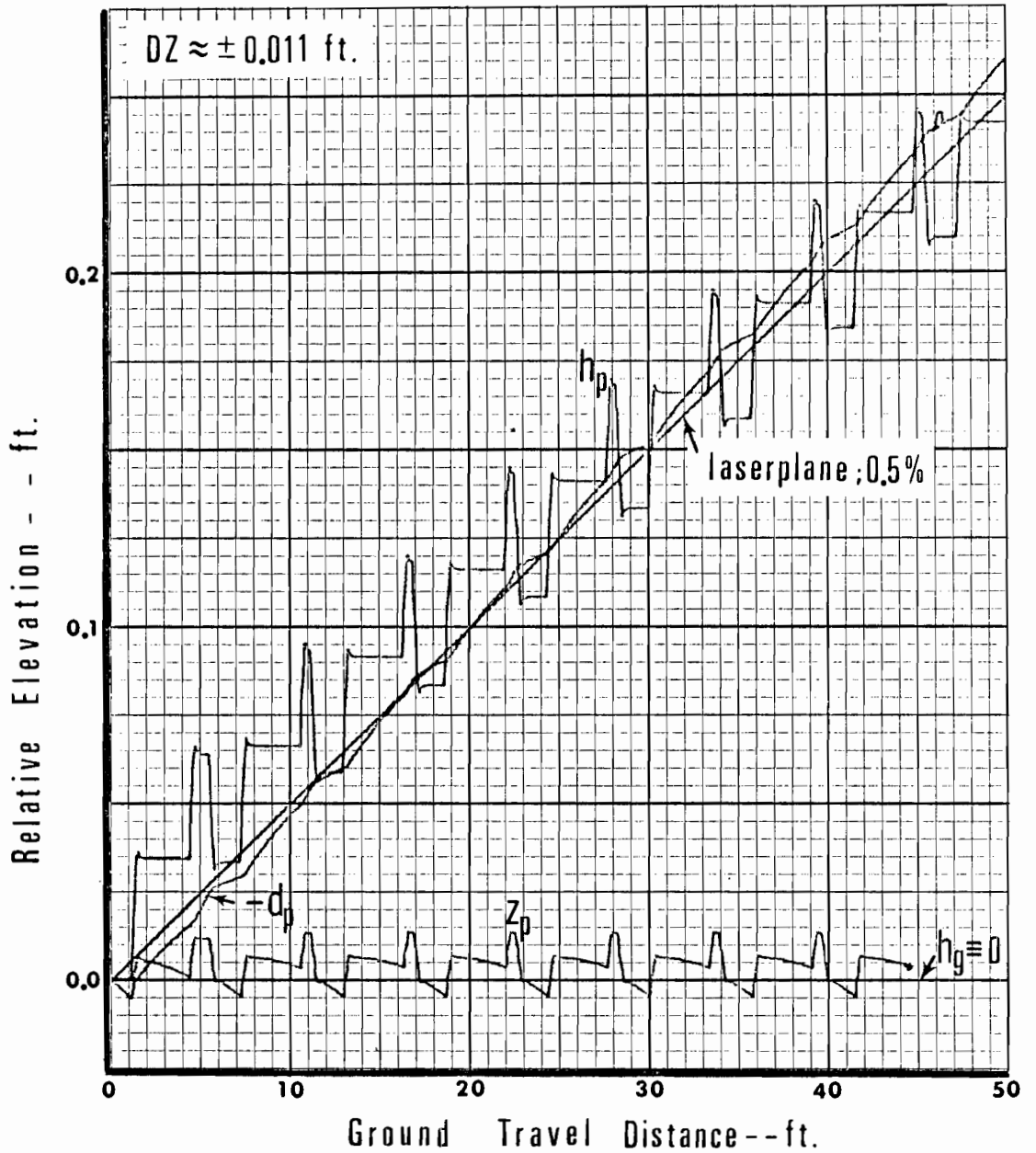


Fig. 40. Dead-zone on-off control mode grading accuracy for level land and a 0.5 percent specified gradient;  $DZ \approx \pm 0.011$  ft.;  $x = 7$  ft.

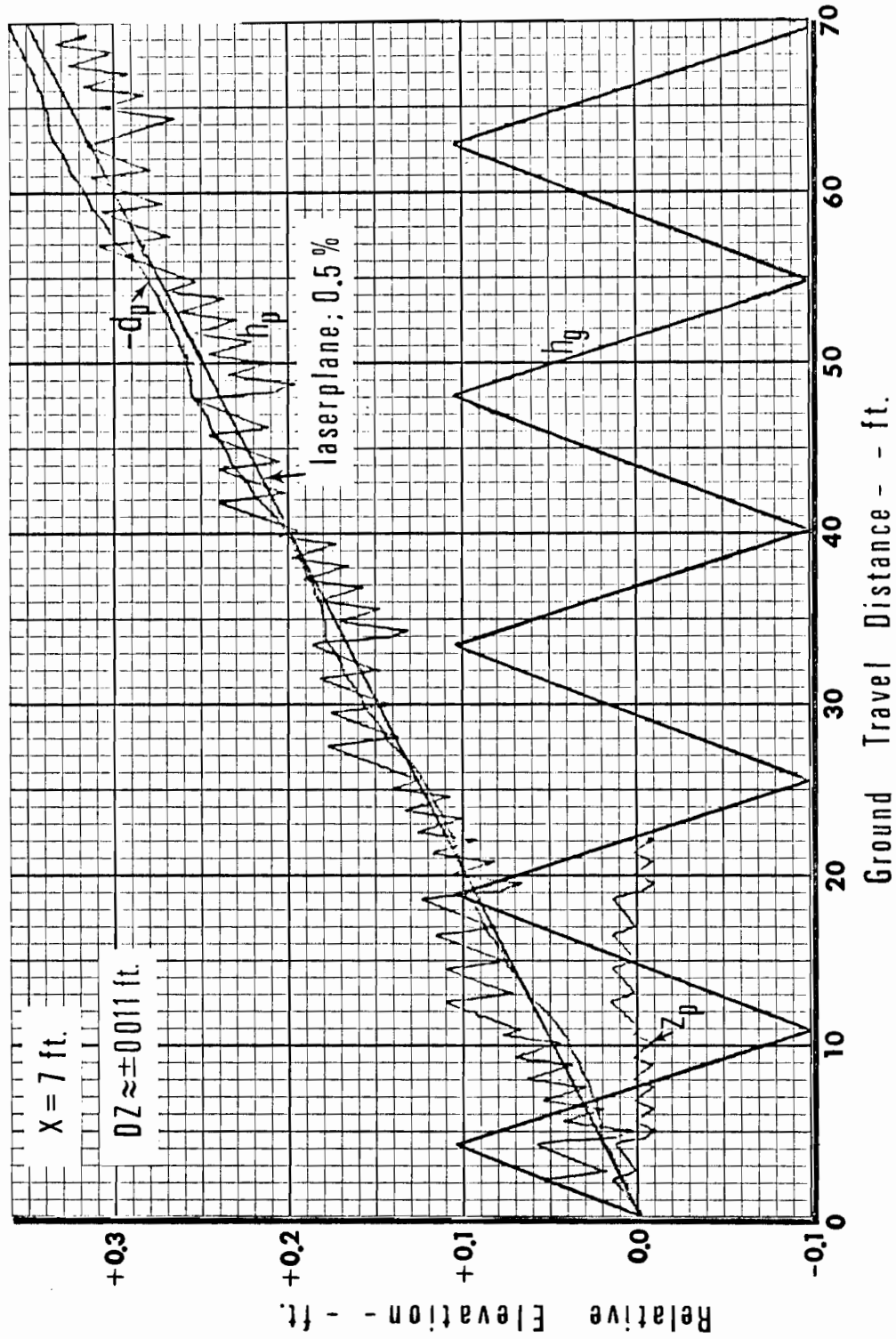


Fig. 41. Dead-zone on-off control mode grading accuracy for a simulated sawtooth ground surface (average land slope zero) and a 0.5 percent specified gradient;  $DZ = \pm 0.011$  ft.;  $x = 7$  ft.

#### 4. Digital On-Off Control Mode

Assuming a "bang-bang" type receiver unit and a digital on-off controller with  $\tau_s = 0.040$  sec.,  $\tau_r = 0.200$  sec. (equation [27] and Figure 27), and  $\dot{h}_c = \pm 0.20$  ft./sec., a simulation run was made to determine the "natural" limit-cycle (for  $h_g \equiv 0$ ). The resulting limit-cycle had the same amplitude as with the simple "bang-bang" mode, but the frequency was only about 0.3 cycle/sec., thus, oscillations in moling depth were greater in magnitude (results not shown). The simulated response for a sine-wave ground surface ( $h_g = \pm 0.1$  ft. at 0.087 cps) revealed considerable oscillation in moling depth, even for the receiver located at the hitch ( $x = 0$ ); Figure 42. It can be seen that the hydraulic cylinder speed would have had to be faster than 0.2 ft./sec. in order to completely cancel out the effects of the ground surface variations.

#### 5. Proportional Control Mode

Only two simulation runs were made for the proportional control mode to illustrate its characteristic performance. The following control system parameters were used:  $K_c = 2 \frac{\text{ft./sec.}}{\text{ft.}}$ ,  $z_{pl} = \pm 0.1$  ft. (equation [29]), and a servo-valve time lag of  $\tau_L = 0.10$  sec. (Figure 28). Figure 43 shows the simulated response for a sine-wave ground surface ( $h_g = \pm 0.1$  ft. at 0.050 cps, and average of zero), and 0.5 percent desired drain grade, with the receiver located at  $x = 4$  and 7 ft. Although the control accuracy meets the minimum requirements, performance could have been improved with a higher gain  $K_c$  and/or a shorter time lag  $\tau_L$  in the hydraulics. It should be noted that as  $K_c$  increases, the proportional

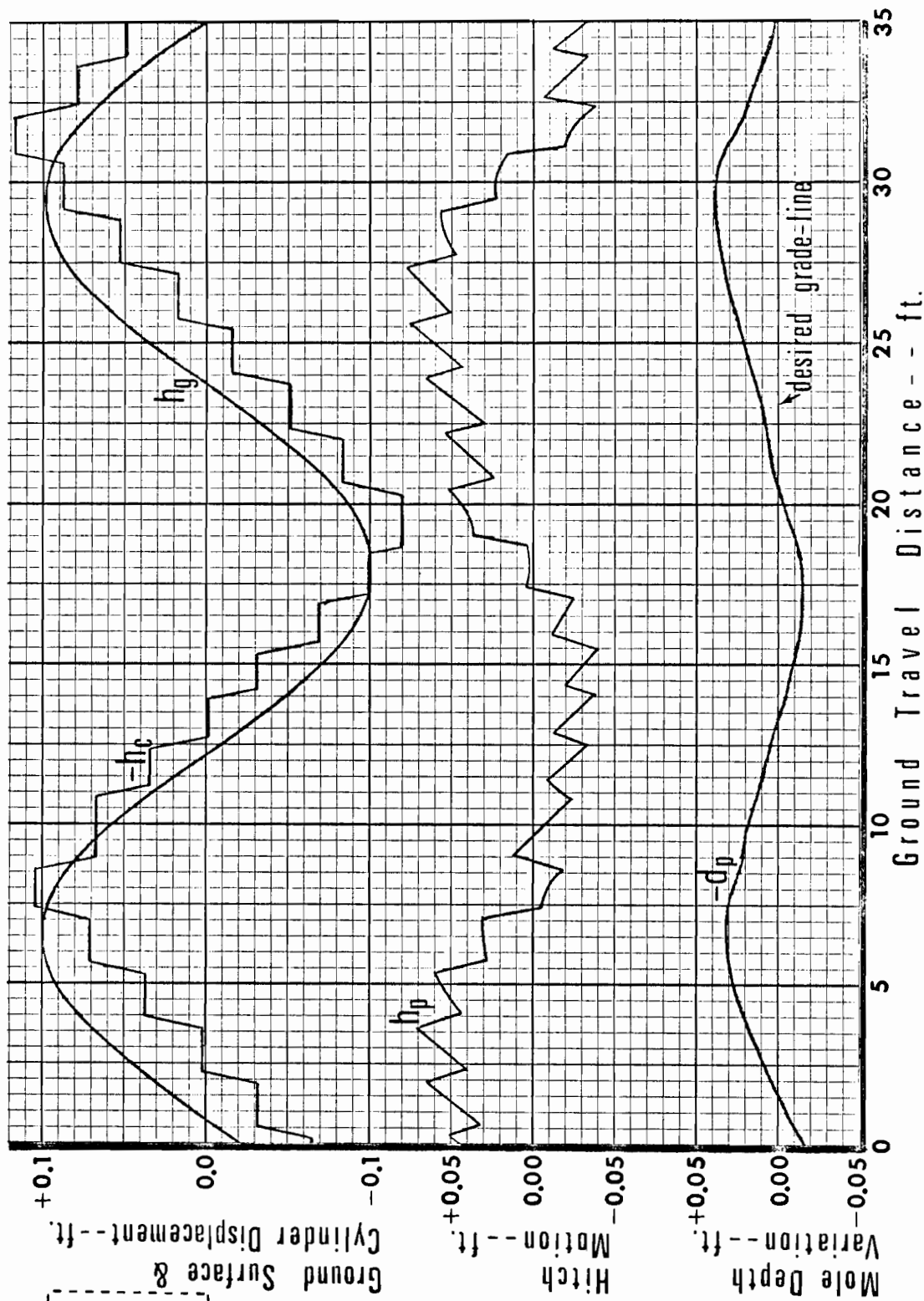


Fig. 42. Digital on-off control mode grading accuracy for a simulated sine-wave ground surface.



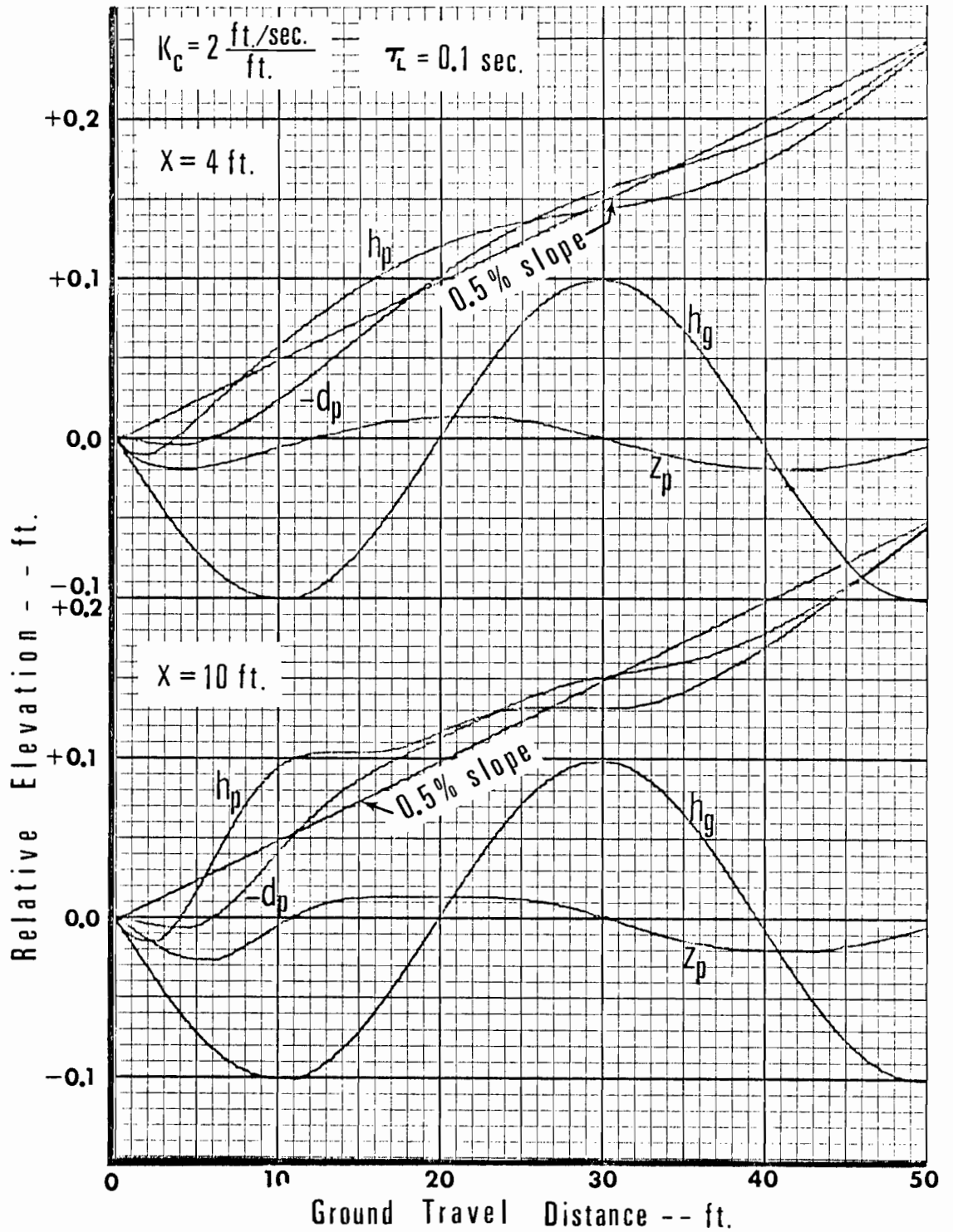


Fig. 43. Proportional control mode grading accuracy for a simulated sine-wave ground surface and a 0.5 percent specified gradient;  $x = 4$  and  $10 \text{ ft.}$

control mode is approximated by the dead-zone on-off mode, and for still higher gain by the simple on-off (bang-bang) mode.

#### IV. DISCUSSION

The computer simulation of the mole plow and laser feedback control system offers an excellent method of analysis. Various combinations of system parameters can be easily tried and quickly evaluated to optimize performance. To accomplish this using field tests alone would be formidable indeed, because of the random variation often found in the field conditions, soils, etc., and the numerous combinations of system parameters to be evaluated, not to mention the cost, time, and labor involved. The computer simulation does not eliminate the need for field evaluation of a proposed control system, but can reduce the number of parameter combinations to be tested under natural field conditions.

The modeling and simulation of the drainage plow presented herein involved conducting a few field tests with a prototype to determine basic performance parameters, such as the effective steady-state gain factor  $\frac{1}{r}$ , and the "settling time" or distance (Appendix B). The relationship between draft and plowing depth was also determined from simplified field tests (Appendix C). The precise relationship was not required to match computed and field response. In the computer circuit, the value of the damping coefficient C could be adjusted to give the best fit to field data. The closeness of the fit in the simulation (Figures 12, 13, and 14) indicates that the plow model is valid. The model and simulation

circuit could be used to study a proposed plow design if one assumes a soil cutting force distribution on the blade and a settling time (distance) for step changes in hitch height. The values of system parameters determined through this study would provide important guidelines for such a theoretical analysis.

One of the more important and interesting performance parameters of the mole plow is its effective steady-state gain factor  $\frac{1}{r}$ . It was this parameter that made the selection of an optimum laser receiver mounting location difficult for various types of ground surfaces. For some blade designs of recently introduced plows, such as the "Badger Minor" plow (Figure 3) which has a 5.5 inch constant thickness blade with a long inclined frontal surface,  $\frac{1}{r}$  is probably higher than the 1.25 used here, thus causing greater depth change per unit hitch displacement. Drain installation at grades not parallel to the mean ground surface slope is quite difficult. The average drain channel is held closer to specified grade where the receiver is mounted slightly forward of the plow blade, but the moling depth has a tendency to oscillate and the relative stability of the control system can become very poor ("hunts" excessively) for critical ground surface irregularities. A compromise is to move the receiver farther forward on the plow beam to insure better relative control stability.

The simulation results suggest that alternative means for mechanical adjustment of moling depth should be considered, such as the adjustment of the blade angle with the beam. Field test results plotted to illustrate this concept are shown in Figure 50 (Appendix B). This technique has been used in one form or another on several drainage plows, for example,

the Saveson mole plow shown in Figure 58 (Appendix C). Although a 1:1 ratio between the effective blade length and moling depth is obtained with this method for a uniform ground surface, the required angle adjustment is somewhat nonlinear (Figure 50, Appendix B). This non-linearity may not create a feedback control problem, provided the receiver is not located too far forward on the beam, but other disadvantages rule out this method of depth regulation. The principal disadvantage is the relative horizontal motion between the hitch H and the mole M as a function of the angle adjustment. Such relative motion creates acceleration and velocity changes in the direction of forward travel of the mole, thus causing "surges" in the soil resistance force  $R_H$ . This phenomenon was one of the major problems found with the Saveson mole plow [Fouss (1961)]. Thus, it was concluded that hitch height adjustment for a long-beamed plow was a better method of depth control. The kinematic design of the hitch positioning mechanism is quite important, however, to minimize the horizontal motion of the mole relative to the tractor.

Simulation showed that a simple on-off (bang-bang) receiver-controller with a 1.0 to 1.5 cycles/sec. natural limit-cycle, and 0.20 ft./sec. maximum vertical hitch velocity, provided stable control operation where the receiver was at least 1.5 ft. forward of the mole. The simulation further showed, for the bang-bang control mode, that 1.5 ft. forward of the mole ( $x = 10$  ft.) is about an optimum location for the laser receiver (Figures 37 and 38). A receiver-controller unit with a small dead-zone should be located somewhat farther forward on the plow beam than a bang-bang one to maintain nearly equal control over the moling

depth. The hydraulic cylinder speed  $\dot{h}_c$  should be set high enough to "keep up" with the maximum expected ground profile changes, but not so high as to cause excessive "hunting" and poor relative stability. It may be possible to adjust the upward and downward velocity of hitch motion to different values (for example,  $\dot{h}_c = +0.2$  and  $-0.3$  ft./sec.) such that, the natural limit-cycling would cause the moling depth to oscillate about the desired depth rather than slightly shallow. However, a higher speed in the downward direction may cause a tendency to "hunt" excessively for some ground surface inputs. This modification of the control mode, if used, would need to be checked out thoroughly.

The simulation study was based on the use of ground surface profiles described by sine-waves, saw-tooth shapes, ramps, and combinations of these, none of which are found exactly in the normal field topography. Although experience has shown that using standard inputs in control system design is successful in most cases,<sup>1/</sup> it is recommended that the simulated plow and control system be subjected to some random ground surfaces to further study the system performance. Variations in the magnitude of the soil resistance force  $R$  caused by soil structure changes, or other factors, were neglected in this study. It is reasonable to expect that the variability of  $R$  during forward travel at a given moling depth, will cause significant unbalanced moments about the hitch which in turn will cause the moling depth to fluctuate. This area should also receive further study. However, it is encouraging that the simulation shows the optimum receiver location is close to the mole rather than the hitch, thus such

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<sup>1/</sup> Doebelin (1962, p. 159).

fluctuations in moling depth, even if small, will be detected by the receiver unit.

For hitch height regulation as a means of depth control, the simulation results further indicate that using two laser receivers on the plow beam, rather than one, could improve grade control accuracy considerably, especially where specified drain gradient is other than the average ground slope. The proposed relative locations for the two receivers are shown in Figure 44. The proposed function and operation of each receiver is: (a) The front receiver provides the usual feedback control signal to the system hydraulics for regulation of the hitch height; and (b) the rear receiver "senses" errors in moling depth from the desired grade-line and provides periodic step-wise signals to adjust mechanically the height of the front receiver with respect to its mounting plate. This will result in limit-cycling the hitch about an average line of travel so that the mole channel is formed at the specified grade parallel to the projected laserplane reference. It is recommended that this dual control concept be evaluated by computer simulation in future studies to determine the best control mode of the front receiver height by the rear one (for example, digital on-off) and the optimum relative locations for the two receivers on the plow beam.

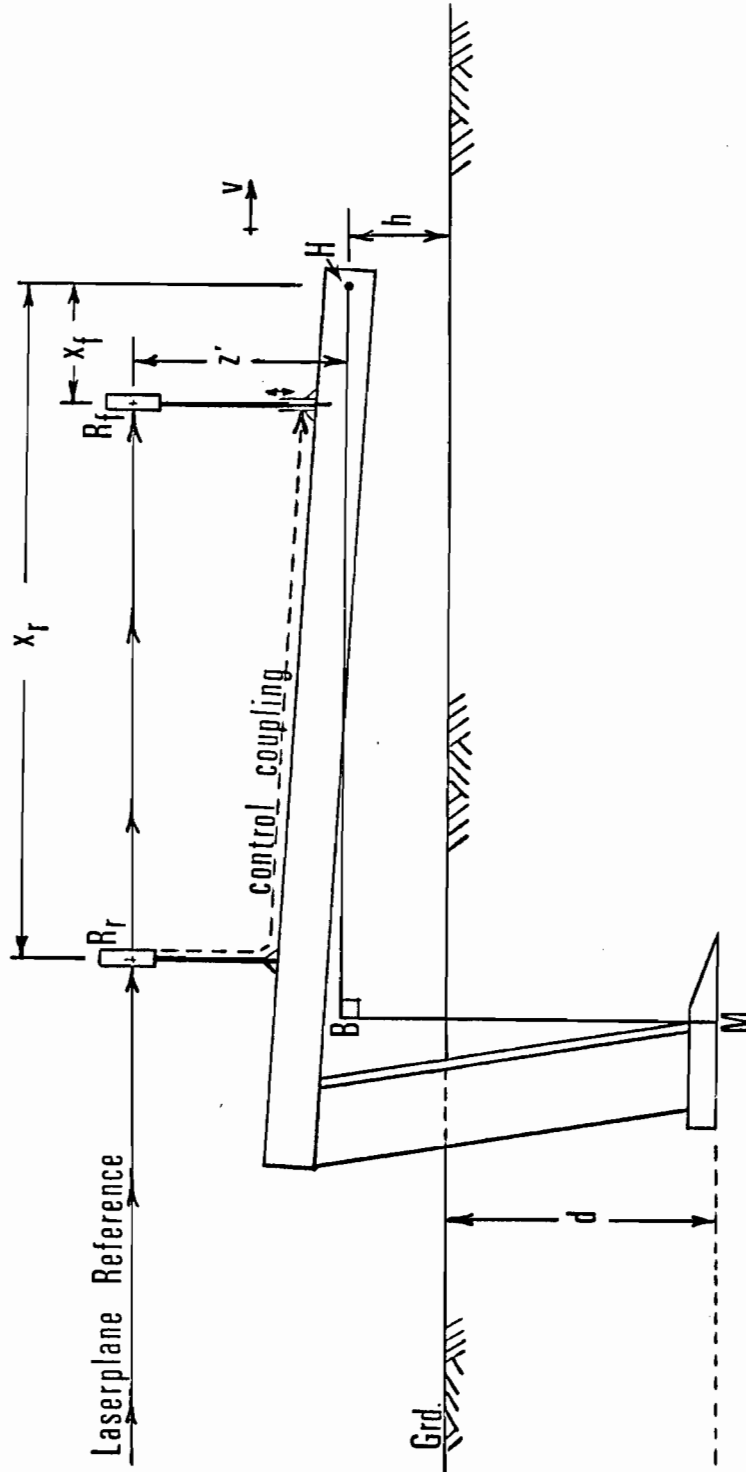


Fig. 44. Concept of dual laser beam receivers on the mole plow for feedback depth and grade control;  $R_f$  controls  $h$ , and  $R_r$  controls  $z''$ .



APPENDIX A

CHARACTERISTICS OF USDA-ARS TOOL-BAR-MOUNTED,  
FLOATING-BEAM MOLE-DRAIN PLOW

A side-view of the mole plow drawn approximately to scale is given in Figure 45; the plow is shown in operating position with the lift and transport cables slack. [A photograph of this plow is given in Fig. 2, page 8]. The plow can also be operated as a depth gage-wheel type if desired (the wheels are not shown in Fig. 45). For this plow the mole-hitch configuration is adjusted with the large turnbuckle screws which fixes the beam-blade angle  $\beta$ ; angle  $\beta$  is set such that the hitch height is normal (centered vertically) for the average moling depth  $d_0$  desired. [In effect this establishes the value of  $n$  as defined in equation [9], p. 26, and used in equation [12], p. 27]. During field operation the moling depth is controlled by varying the hitch height with the linkage mechanism as illustrated in Figure 46.

The characteristics of the hitch motion, when displaced by the hydraulic actuated hitch linkage mechanism, were determined by testing the prototype. A linear motion transducer for recording hitch motion was devised by using a ten-turn, 10K potentiometer and a pulley-belt drive mechanism. Simultaneously, the voltage applied to the hydraulic solenoid-valve was recorded. Test results are shown in Figures 47 and 48, where it is seen that the time delay in the hydraulic valve system is  $\tau_L = 0.15$  sec. For these tests the hydraulic cylinder velocity was adjusted to  $h_c = \pm 2.4$  to  $2.5$  in./sec.  $\approx \pm 0.20$  ft./sec.; the upward and

Ref.: Figure 2, page 8;  
 { Optional depth-wheels not  
 shown on drawing below. }

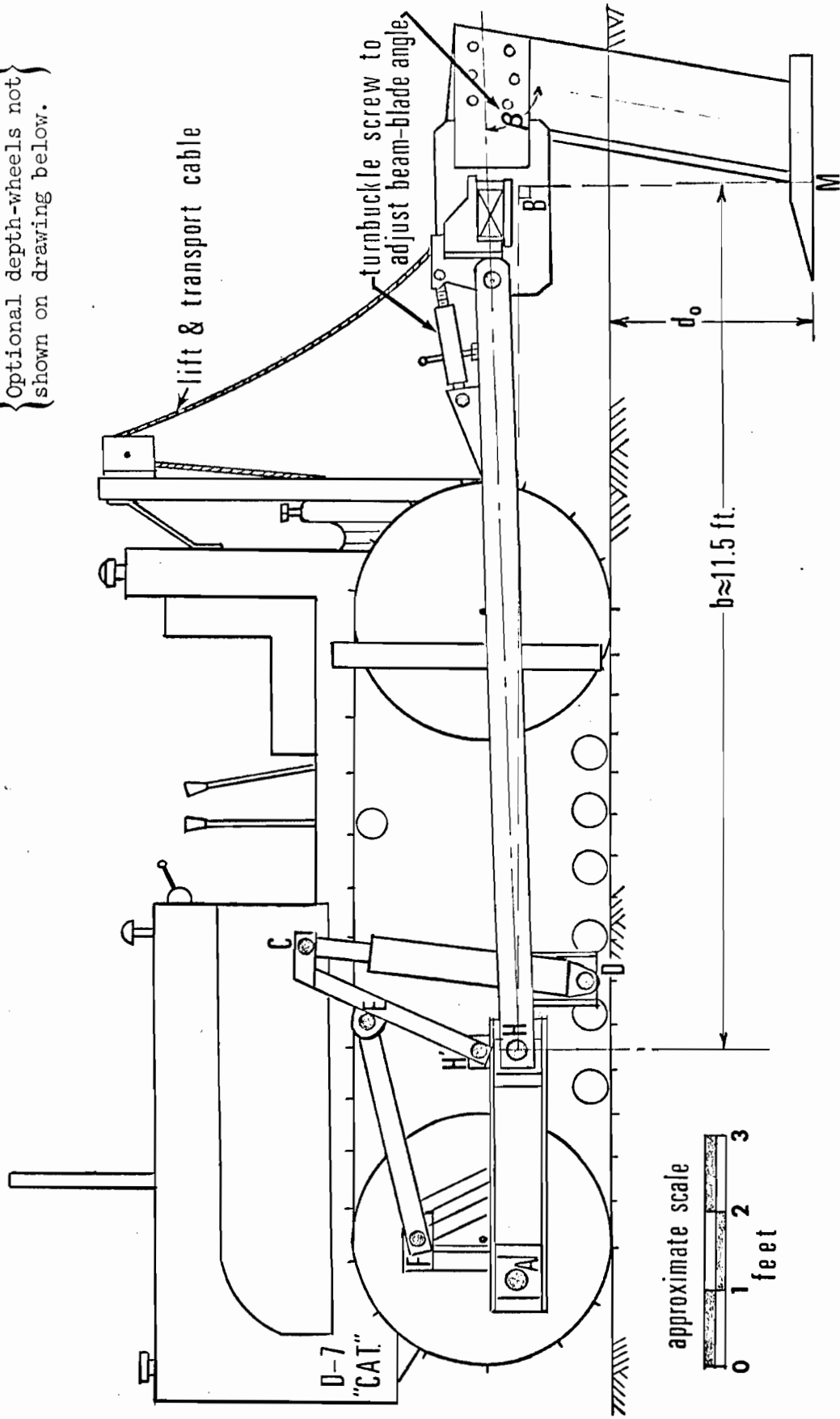


Fig. 45. Side-view of USDA-ARS, tool-bar-mounted, floating-beam mole plow.

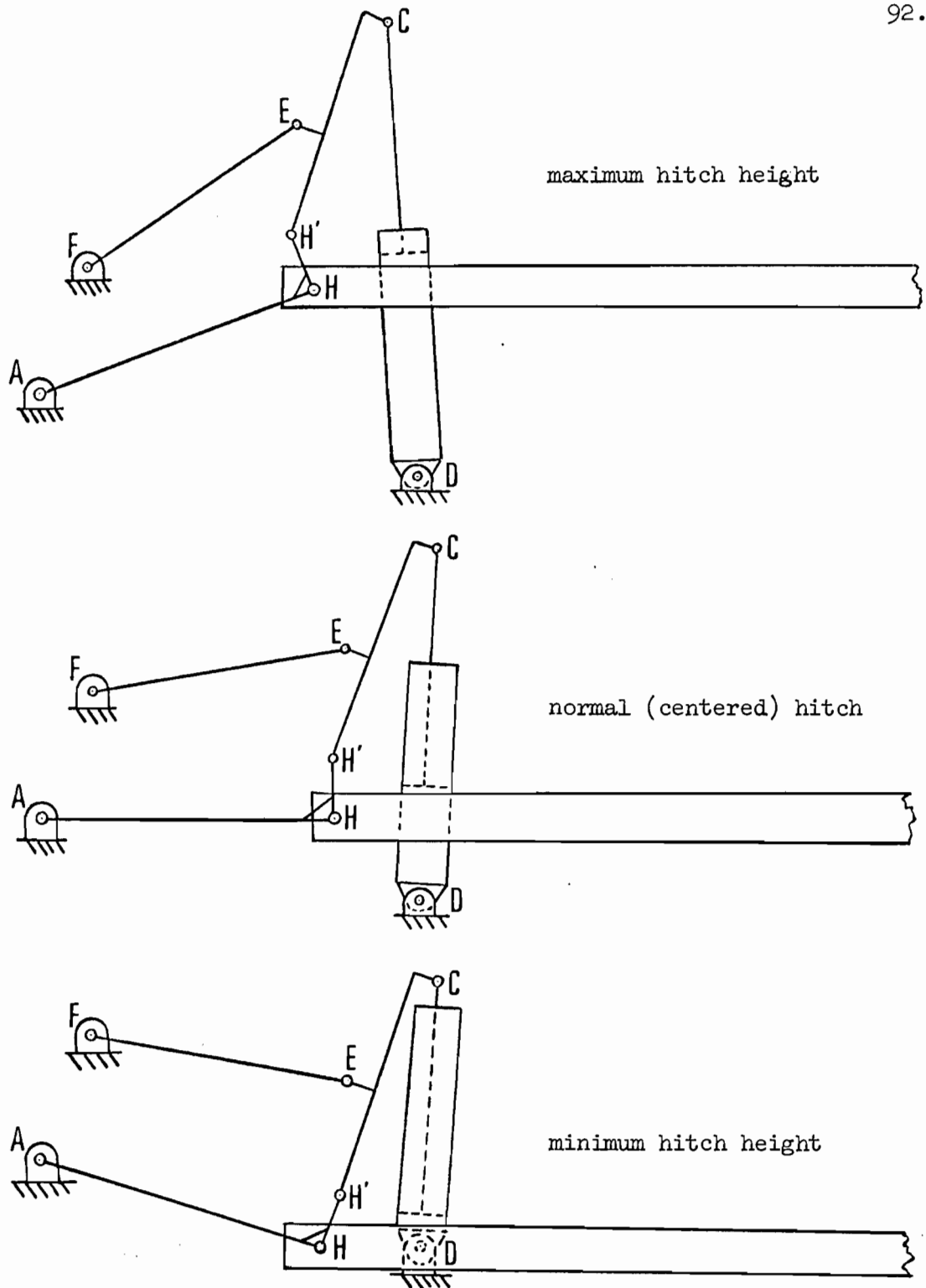


Fig. 46. Kinematic motion of hitch control linkage for USDA-ARS floating-beam mole plow.

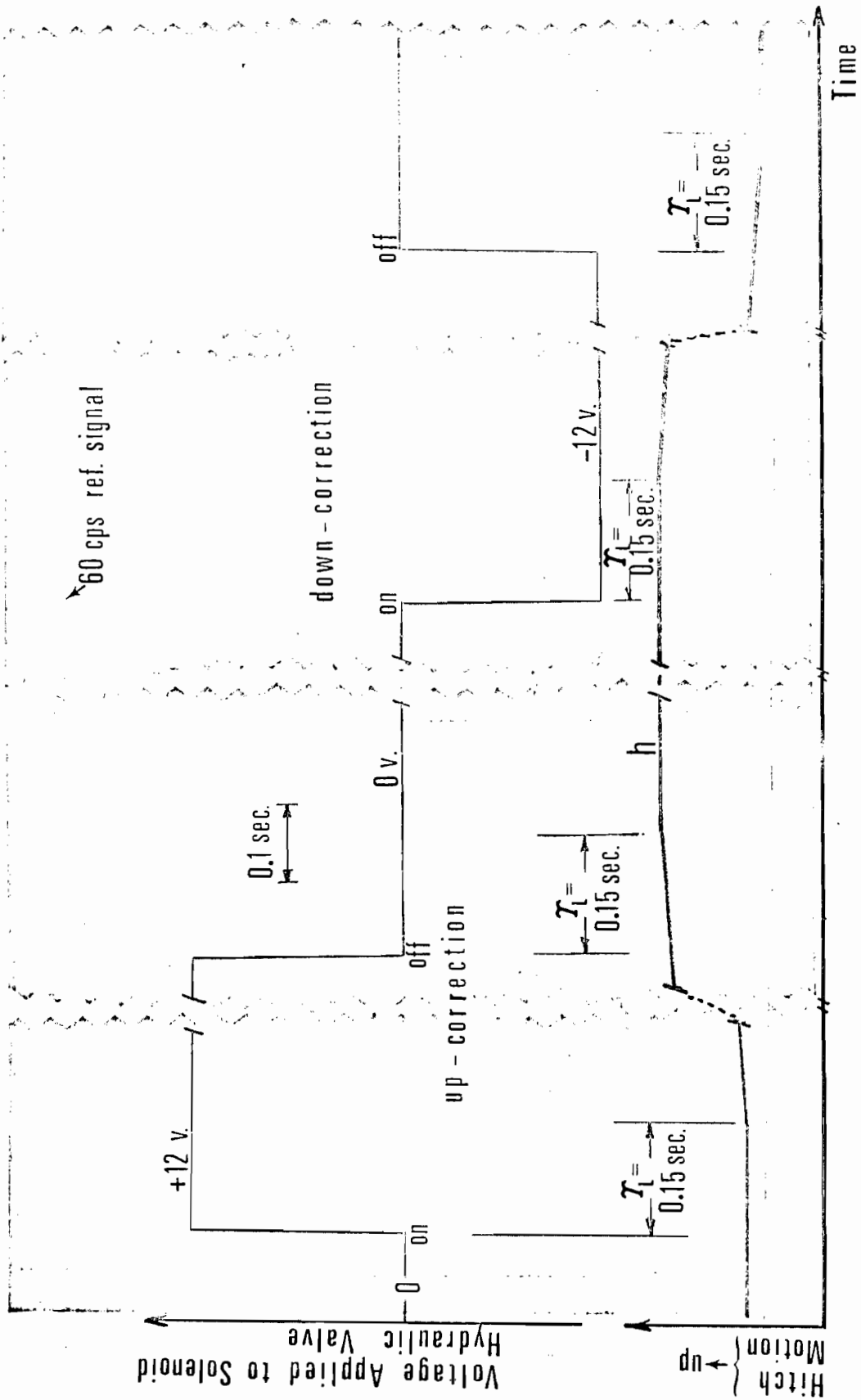


Fig. 47. Strip chart record of applied voltage to hydraulic solenoid valve and corresponding time-delayed ( $t - \tau_L$ ) hitch motion response.

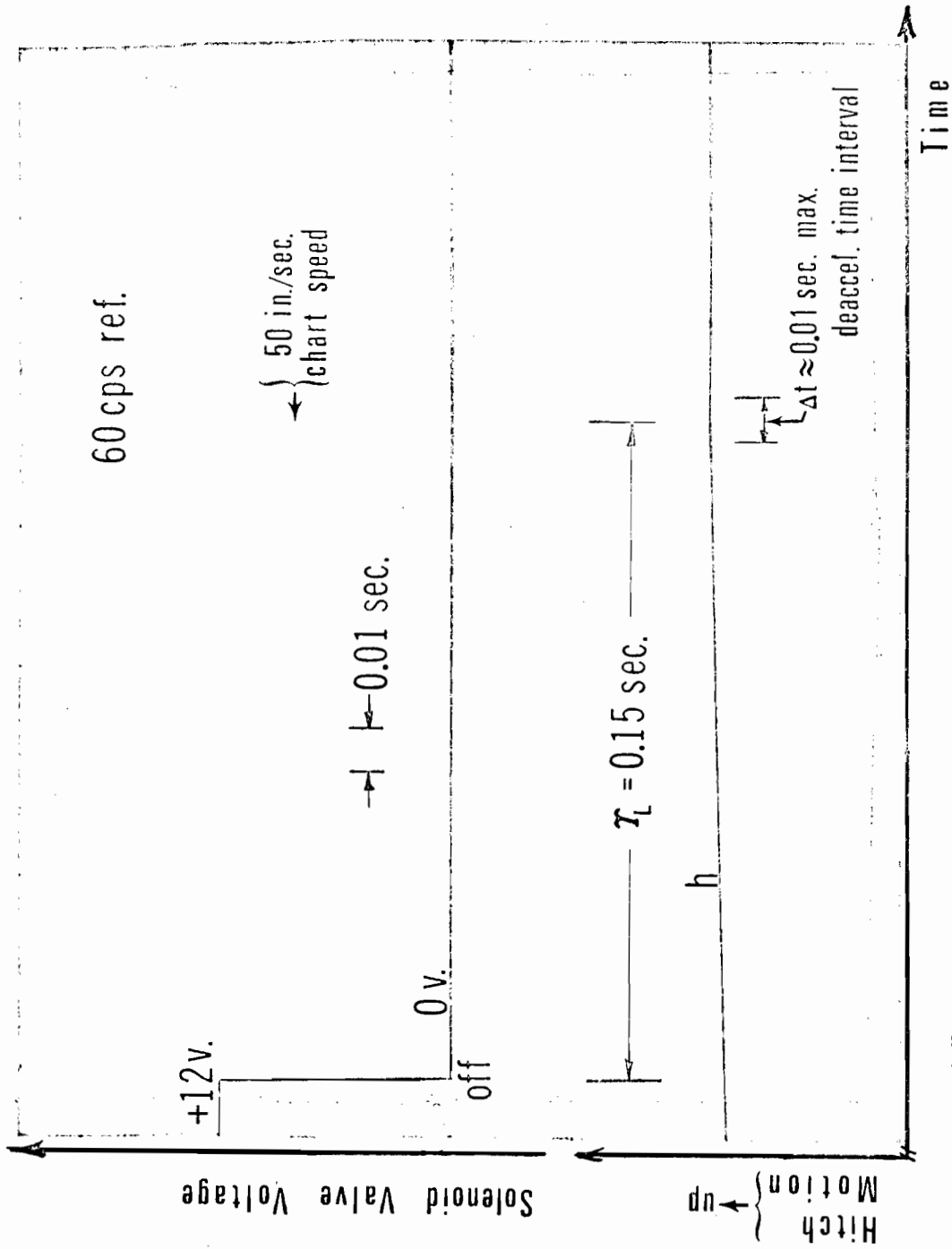


Fig. 48. High speed strip chart recording of hitch motion dynamics as motion is stopped following closure of solenoid valve.

downward speeds were nearly the same (graphical results not shown). In Figure 48 it can be seen that the maximum time interval for deacceleration of the hitch velocity from +0.20 ft./sec.  $\rightarrow$  0 is  $\Delta t \approx 0.01$  sec. ( $\Delta t$  for acceleration was the same -- not shown). Thus, the peak hitch acceleration  $\ddot{h}$  can be estimated from the average velocity over one-half the time interval; that is,

$$\ddot{h}_{\text{peak}} \approx \frac{\frac{h_{\text{max}}/2}{2}}{\Delta t/2} = \frac{0.20/4}{0.01/2} = 10.0 \text{ ft./sec.}^2$$

The plow weight ( $mg \approx 4000$  lbs.) had been determined previously by weighing on a truck scale. The measured weight checked closely with that estimated from calculated weight for each component or part summed (calculations not shown). The location of the plow center-of-gravity G ( $a = 8.92$  ft. back from the hitch H) was determined analytically from the known shape and calculated weight of each component or part. The theoretical location for G was confirmed by the computed and observed reaction force near zero in the large turnbuckle screw, when the plow was held in the transport position with the cables; Fig. 45 (the corrugated draintube laying device was attached).

The mass moment-of-inertia ( $J_G = 1,458$  ft.-lb.-sec.<sup>2</sup>) was determined analytically by use of the "parallel-axis theorem"; this method was used since most of the individual parts on the plow were made up of simple shapes, for which the c.g. was known or could be easily determined analytically, and for which the moment-of-inertia could be calculated by standard methods of engineering mechanics (calculations not shown).

APPENDIX B



DETERMINING RESPONSE PARAMETERS OF  
USDA-ARS FLOATING-BEAM MOLE-DRAIN PLOW

Steady-state and transient response field tests were specifically conducted to evaluate the numerical values of: (a) The fraction  $\underline{r}$  of the moling depth where the draft force acts on the blade; (b) the distance  $\underline{n}$  below the hitch where the draft acts; and (c) the effective plow beam length  $\underline{b}$ . Referring to page 26, it was assumed that (equation [9])

$$n = rd_s + h_s ,$$

where  $n$  is constant for all corresponding  $d_s$  and  $h_s$  at a given plow configuration, and for some specified range of moling depth. It follows that:

$$d_s = -\frac{1}{r} h_s + \frac{n}{r} .$$

Field test results confirm this relationship is a good approximation for use in the analysis, and yield both  $r$  and  $n$  as shown in Figure 49; the field data are given in Table 3. It can be seen that  $-\frac{1}{r} = -1.25$  [ $r = 0.8$ ] holds for all plow configurations tested, but  $n$  varied with the turnbuckle setting (Fig. 45, p. 91) as expected; for the computer simulation study (Chapter II), a typical value of  $n = 2.82$  ft. was used and was applicable for the range  $1.5 \leq d \leq 3.0$  ft. [Fig. 49; data points ⊙]. These same field data are plotted in Figure 50 to illustrate the possible control of moling depth by adjustment of the turnbuckle screws which changes the beam-blade angle  $\beta$  (Fig. 45).

TABLE 3. Field Test Data of Corresponding Steady-State Hitch Height ( $h_s$ ) and Moling Depth ( $d_s$ ).

Turnbuckle Screw Setting  (no. threads) <sub>1/</sub>	Hitch Height $h_s$  (ft.)	Moling Depth $d_s$  (ft.) <sub>2/</sub>	Data Plotting Designation  [Fig. 49]
22	1.17	2.63	} □
-do-	1.67	1.96	
23	1.17	2.27	} △
-do-	1.67	1.71	
24	0.67	2.73	} ○
-do-	1.17	1.96	
-do-	1.67	1.50	
25	1.29	1.45	} + <sub>3/</sub>
-do-	0.79	2.20	
-do-	1.29	1.55	
-do-	0.83	2.1 - 2.2	
-do-	1.42	1.50	
-do-	0.87	2.15	

<sub>1/</sub>Ref.: Fig. 45, p. 91; Setting indicates number of exposed screw threads.

<sub>2/</sub>Mole depth was measured by use of an inverted "T-probe" made of strap iron, which was inserted into the soil fracture left by the blade. A wooden "arch-frame" providing a 6-inch ground clearance was placed across the travel path to establish a reference on undisturbed ground.

<sub>3/</sub>Data for steady-state conditions before and after transient response in tests number 1 to 5 covered in the next section of Appendix B.

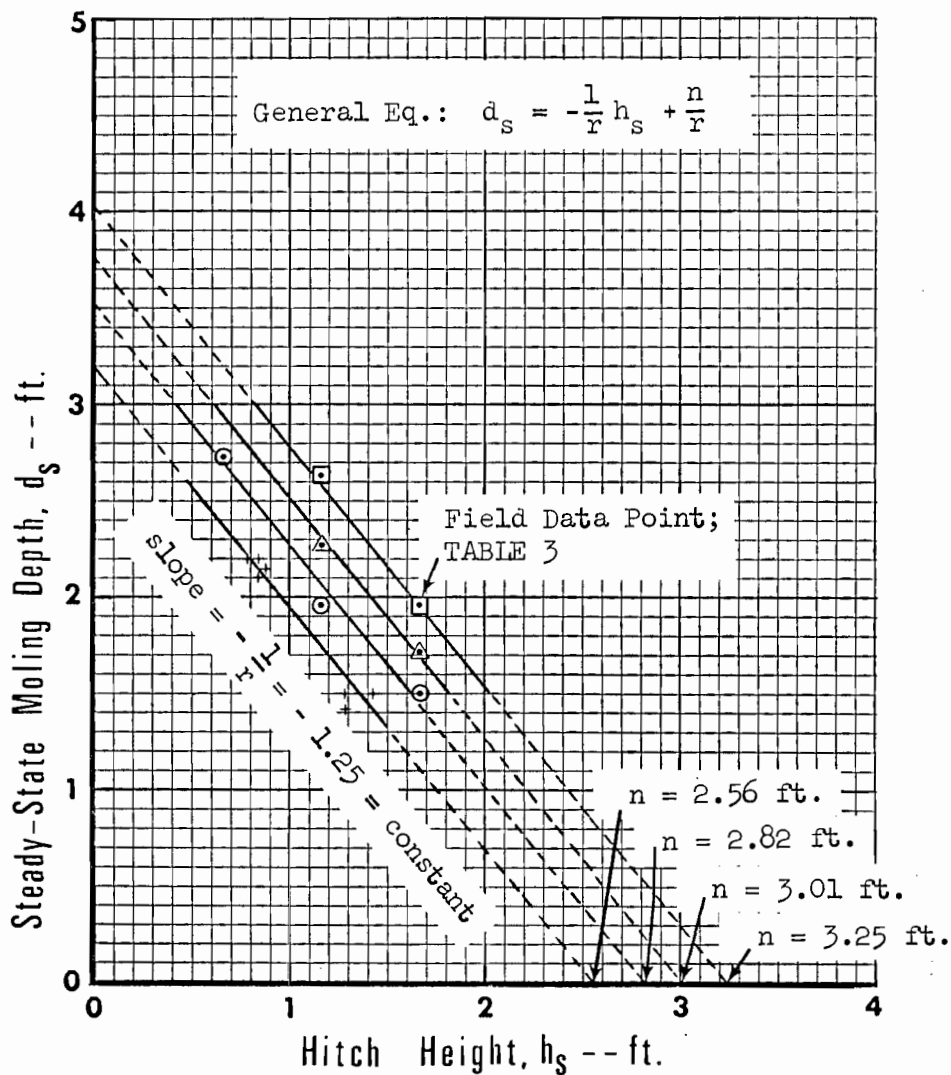


Fig. 49. Steady-state moling depth ( $d_s$ ) versus the corresponding steady-state hitch height ( $h_s$ ) for various mole plow configurations.

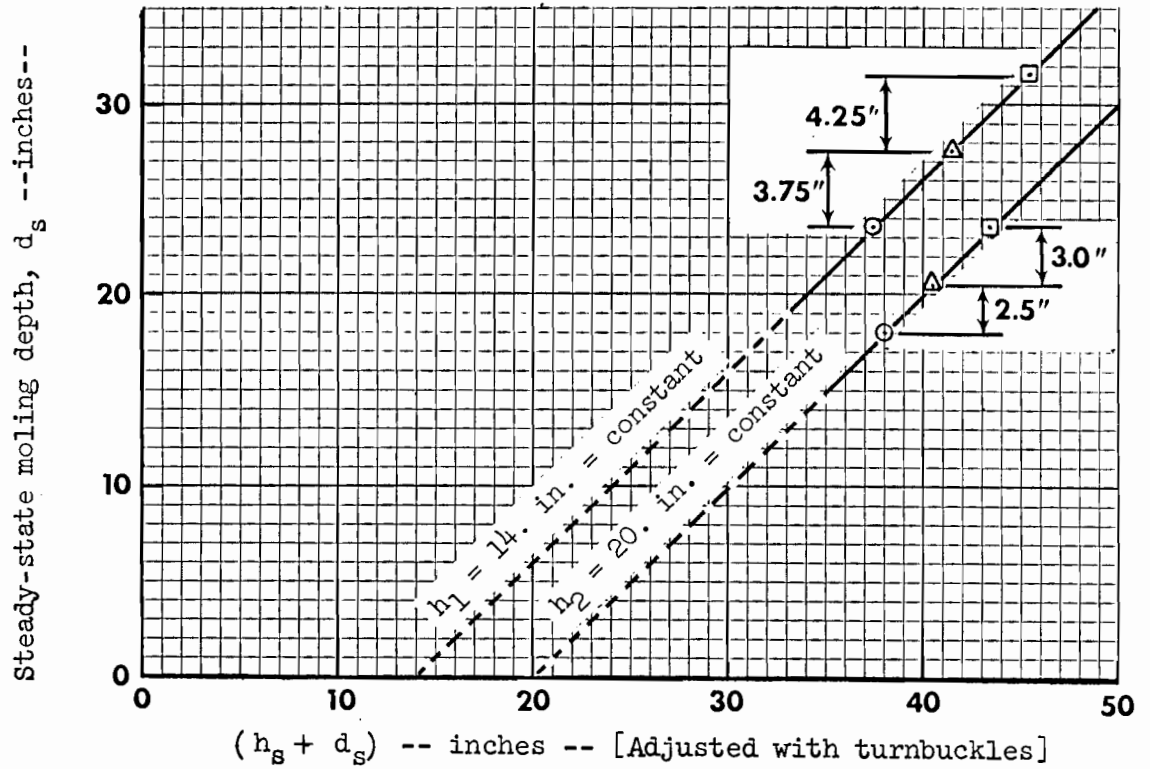
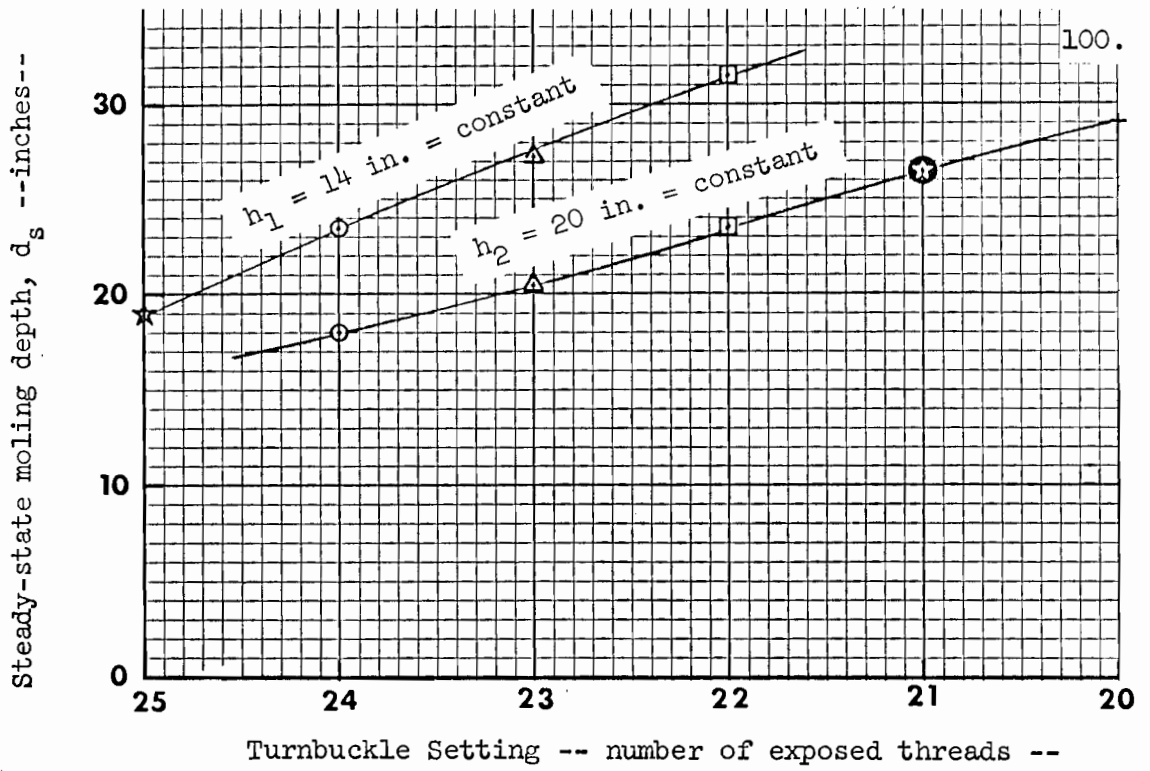


Fig. 50. Steady-state moling depth ( $d_s$ ) versus turnbuckle setting to adjust the beam-blade angle.

Field tests were conducted on The Ohio State University Farm (Brookston silty clay soil) to determine the transient response of the USDA-ARS mole plow to step changes in the hitch height. The elevation profile of the ground surface and the bottom of the mole-drain channel for each field run was determined with optical surveying equipment; the bottom of the mole channel was reached with a special invert "T-probe" made of strap iron and 3 feet in length. Five field tests were conducted [example data are included in Table 4 for Field Test No. 5]. The test results are represented graphically in Figures 51 through 55. Test No. 1 was not useful since the hitch was mistakenly raised first rather than lowered (while forward motion was stopped), thus resulting in the particular response shown. Test No. 3 was not used to compare with simulated response because of the large ground surface change at the point where the hitch was moved.

The "effective" beam length for the mole plow ( $b = 11.5$  ft.) was determined by inspection of the graphical representation of the field data. It was noted that the first deviation of the mole channel bottom, caused by a step change in the hitch height, occurred about 11.5 ft. behind the hitch. This dimension is shown on the plow drawing included in Appendix A, p. 91, Fig. 45.

The steady-state operation before and after each change in hitch height also provided additional data for determining the effective steady-state gain factor ( $-\frac{1}{r}$ ). From equation [9], page 26, one can write:

$$rd_s + h_s = n = rd'_s + h'_s .$$

It follows that,

$$\frac{d_s - d'_s}{h_s - h'_s} = -\frac{1}{r} .$$

This factor was computed from the field data for each test as shown in Figures 51-55; the previously determined value of - 1.25 (Fig. 49) was considered satisfactory for use in the simulation study.

The data in Table 5 were interpolated from Figures 52, 54, and 55, for Field Tests No. 2, 4, and 5, respectively, for use in the computer simulation (Chapter II).

TABLE 4. Example Field Test Data from Transient Response Test No. 5  
for A Downward Ramp-Step Change in Hitch Height.

Ground Travel Distance (ft.)	Ground Surface Elevation (ft.) <u>1/</u>	Mole Channel Elevation + 3 ft. (ft.) <u>2/</u>	Remarks	
0+00	10.61	12.11	Test conducted: 11-28-70	
0+10	10.63	12.17		
0+15	10.60	12.11		
0+20	10.63	12.13		
0+24	10.63	12.10		
0+26	10.60	12.11		
0+28	10.58	12.08		
0+30	10.62	12.06		
0+32	10.50	12.00		
0+34	10.55	11.95		
0+36	10.54	11.89		
0+38	10.48	11.80		
0+40	10.53	11.73		← { hitch started down
0+42	10.50	11.70		
0+44	10.49	11.65		← { hitch stopped
0+46	10.46	11.55		
0+48	10.50	11.55		Δh = -0.54ft.
0+50	10.52	11.52		
0+52	10.52	11.53		
0+54	10.49	11.50		
0+58	10.54	11.44		
0+62	10.54	11.38		
0+66	10.50	11.34		
0+70	10.48	11.33		
0+74	10.46	11.32		
0+78	10.46	11.33		
0+82	10.47	11.33		

1/ On undisturbed ground at the inside edge of the crawler tractor track.

2/ Determined with 3-foot long inverted "T-probe".

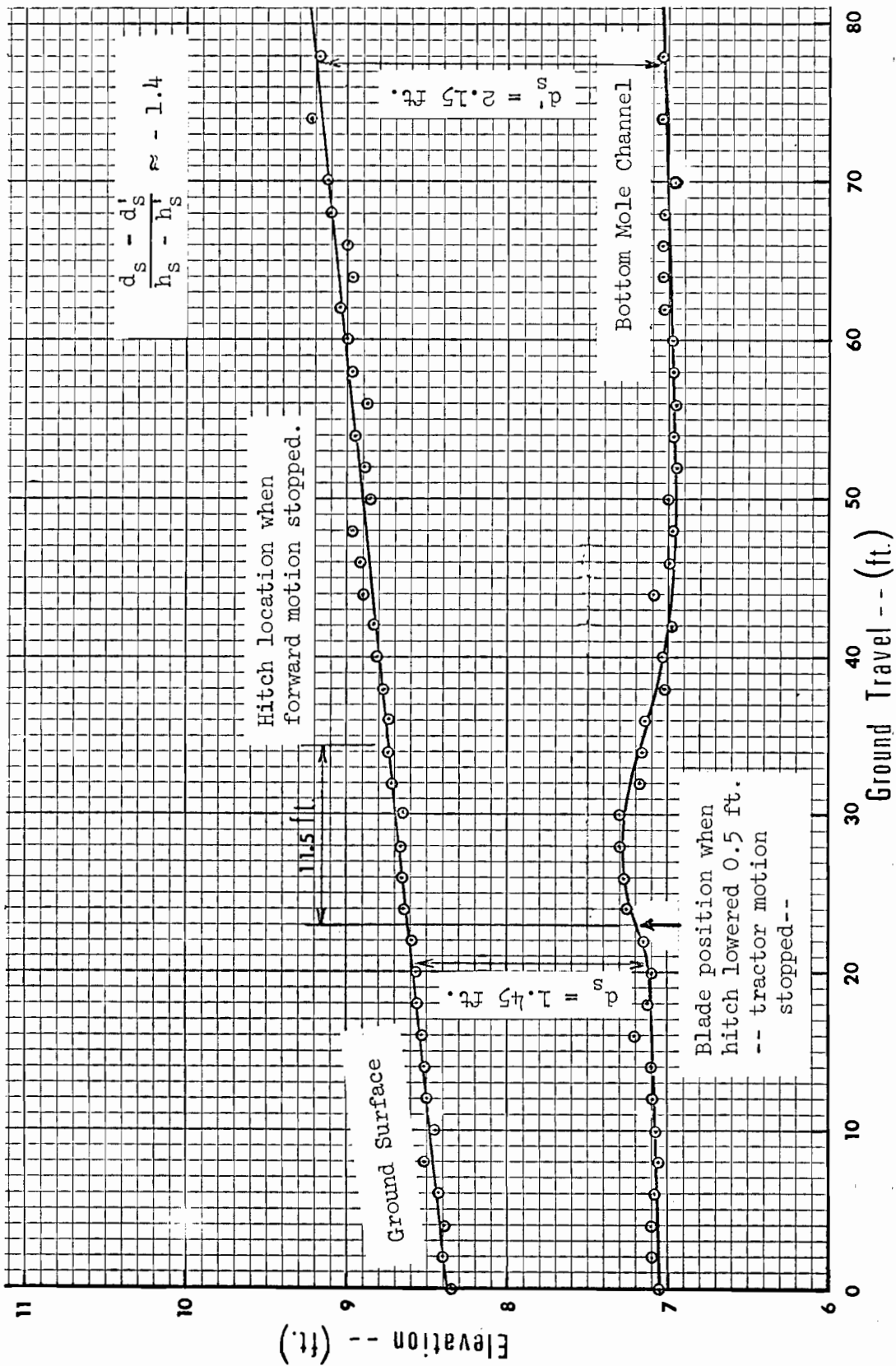


Fig. 51. Field Test No. 1 of mole plow response to a downward step-change of the hitch height.



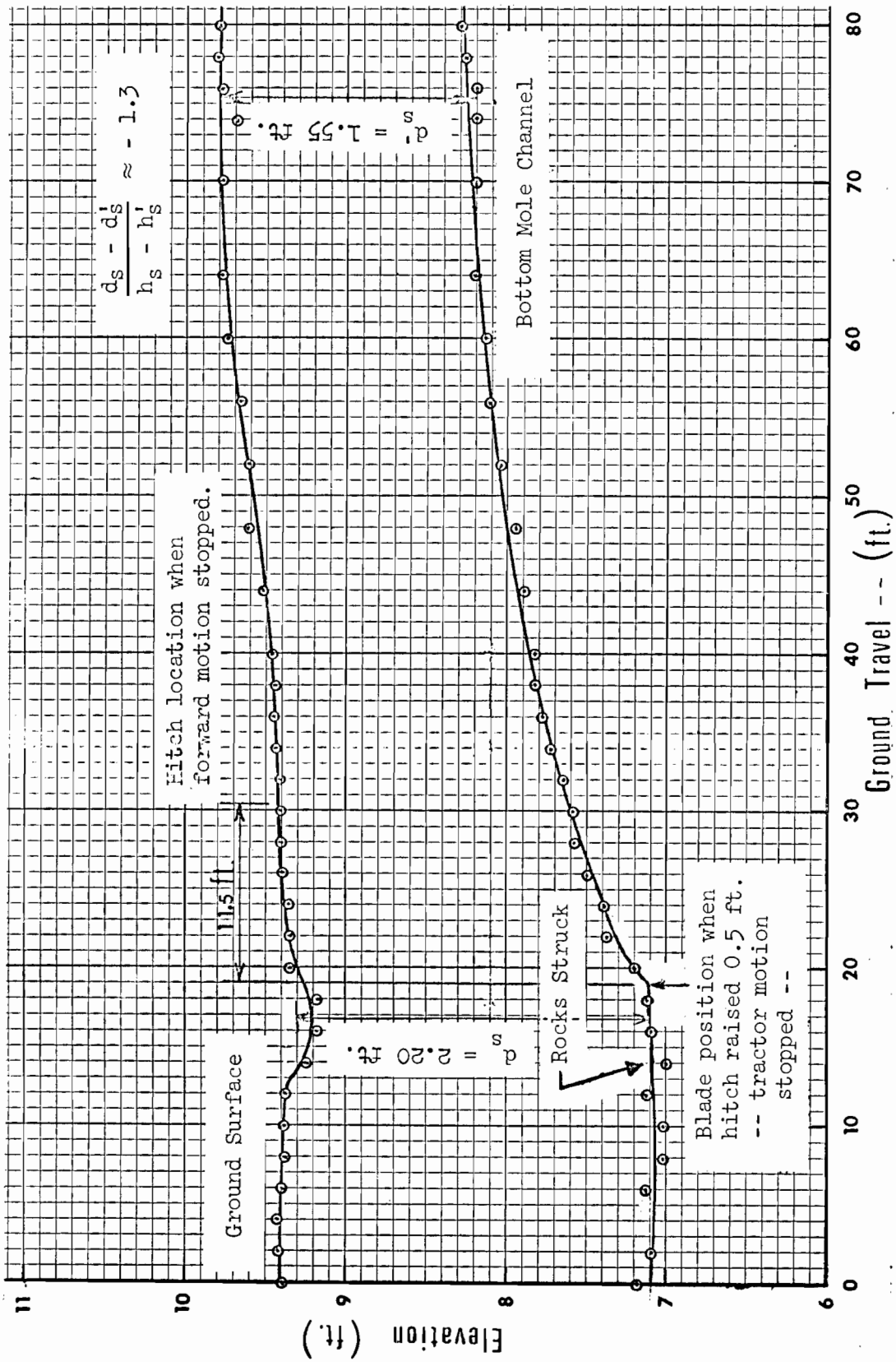


Fig. 52. Field Test No. 2 of mole plow response to an upward step-change of the hitch height.

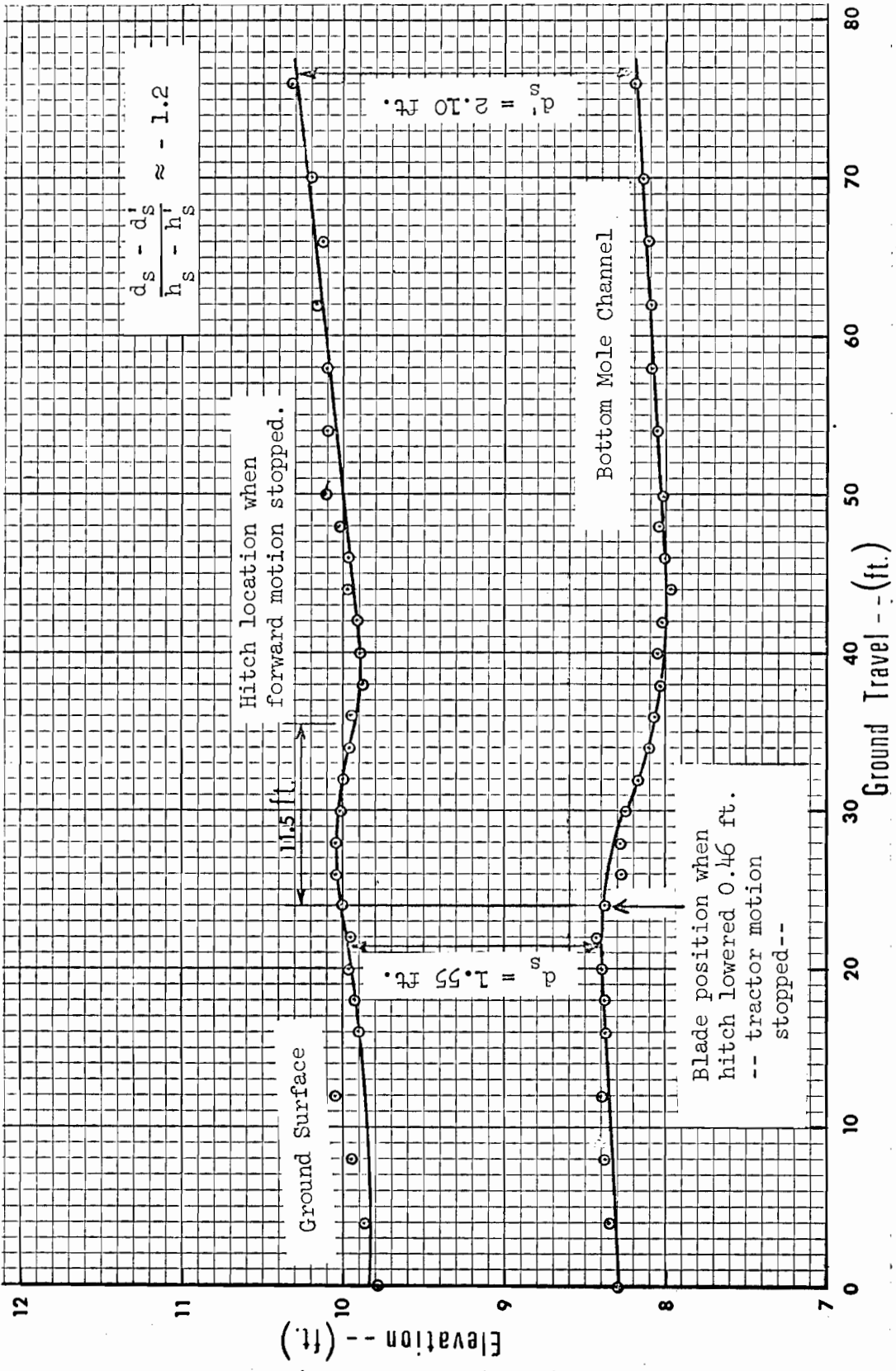


Fig. 53. Field Test No. 3 of mole plow response to a downward step-change of the hitch height.

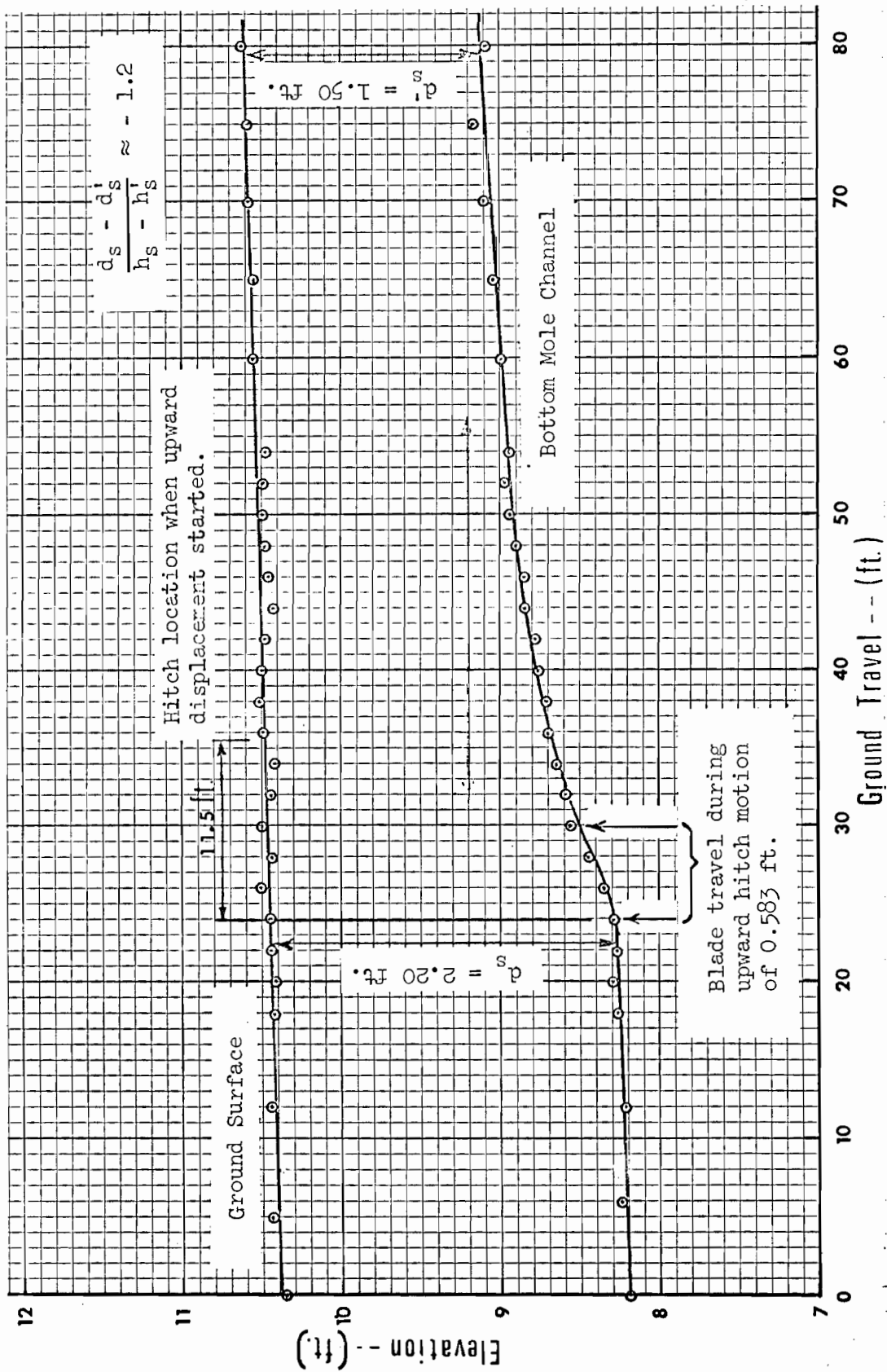


Fig. 54. Field Test No. 4 of mole plow response to a upward ramp-step change of the hitch height. 107.

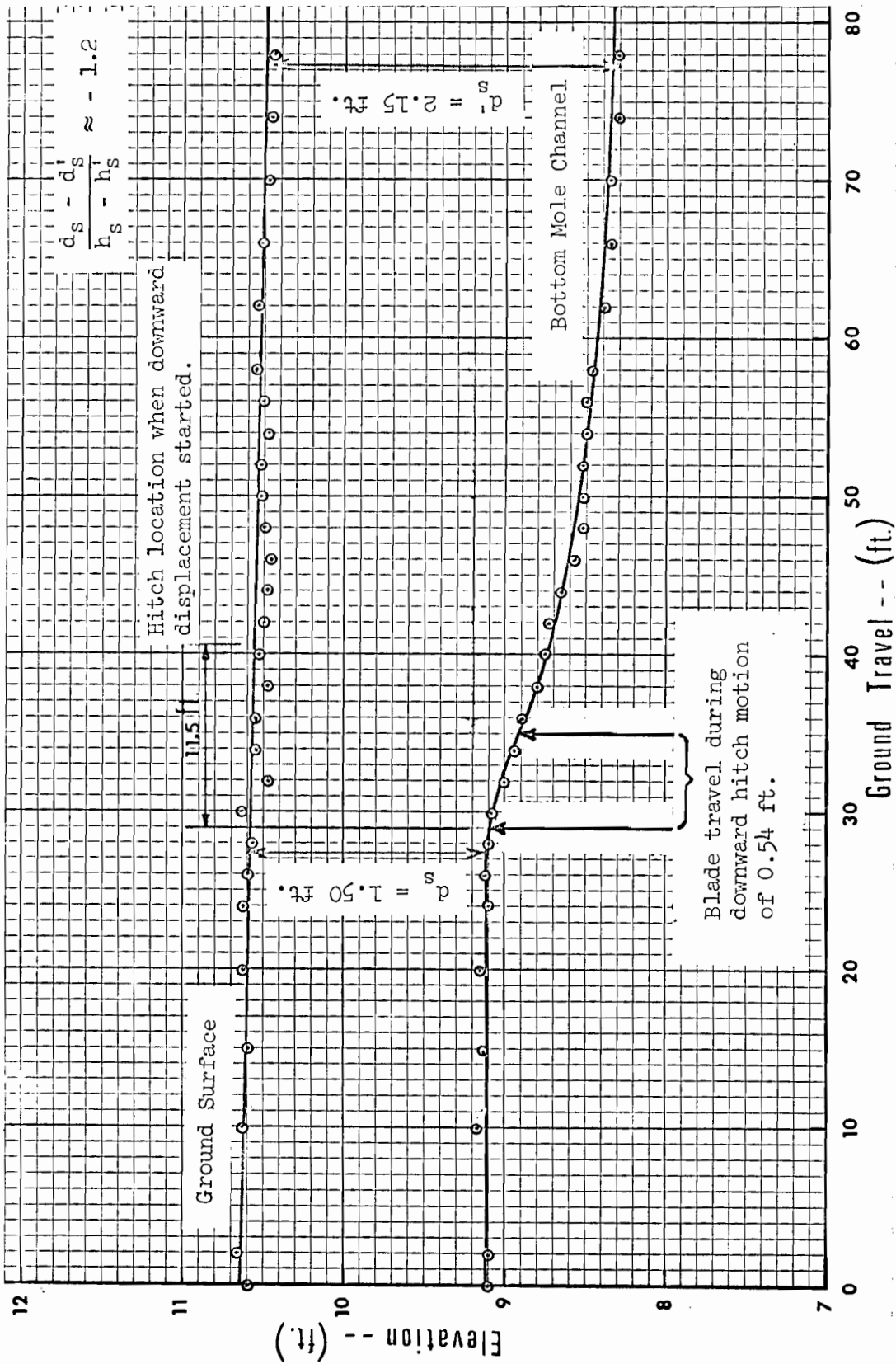


Fig. 55. Field Test No. 5 of mole plow response to a downward ramp-step change of the hitch height. 10

TABLE 5. Data Interpolated From Field Response Tests  
 On USDA-ARS Mole Plow For Comparison With  
 Computer Simulated Response.

Forward Mole Travel  (ft.)	Test # 2 (Fig.52) d  (ft.)	Test # 4 (Fig.54) d  (ft.)	Test # 5 (Fig.55) d  (ft.)
0 <sup>1/</sup>	2.20	2.20	1.50
5	1.95	2.00	1.60
10	1.85	1.82	1.80
15	1.70	1.72	1.90
20	1.65	1.65	2.00
30	1.60	1.55	2.10
40	1.58	1.50	2.15

<sup>1/</sup> At this initial position the mole (M) and hitch (H) are at corresponding steady-state locations, but at 0<sup>+</sup> hitch motion begins to occur if tractor is moving, and has occurred if tractor was stopped.

APPENDIX C

DETERMINING AN EQUATION EXPRESSING DRAFT VERSUS DEPTH  
FOR A MOLE PLOW

The primary purpose of this Appendix is to give simplified methods for field testing of a mole plow, and analysis of the data, to determine approximate draft relationships. Analyses were made for three different mole plows so that the general power function form of the draft equation was confirmed. The particular plows considered were:

- (a) Long-beam mole plow tested by Ede (1958) in England<sup>1/</sup>
- (b) Saveson mole plow<sup>2/</sup>
- (c) USDA-ARS mole plow (Appendix A).

The resulting draft equation developed for each plow is represented graphically in Figure 56. A log-log plot of the draft versus depth data for each plow is shown in Figure 57. The field tests and analysis for the USDA-ARS mole plow were conducted specifically to approximate a draft equation for use in the analog computer simulation study of the plow (Chapters II and III). The analysis details for each plow follow.

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<sup>1/</sup>Ede, A. N. 1958. Innovations in land drainage methods. The Journal of the Institution of British Agricultural Engineers, 14(1).

<sup>2/</sup>Saveson, I. L. Mole drainer and subsoiling plow. (U. S. Patent No. 2,715,286, issued August 16, 1955), U. S. Patent Office.

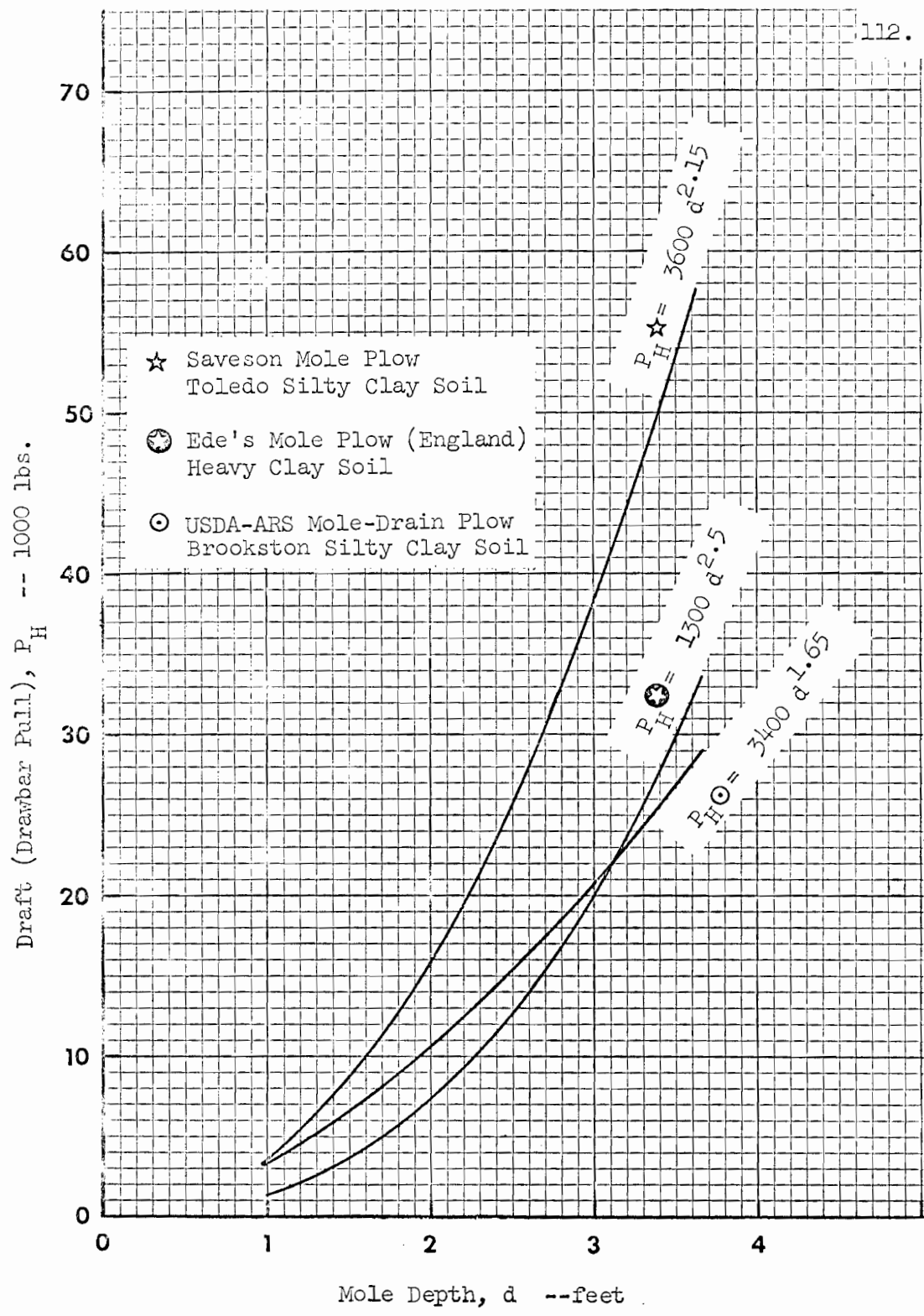


Fig. 56. Draft versus depth relationships for three mole plows



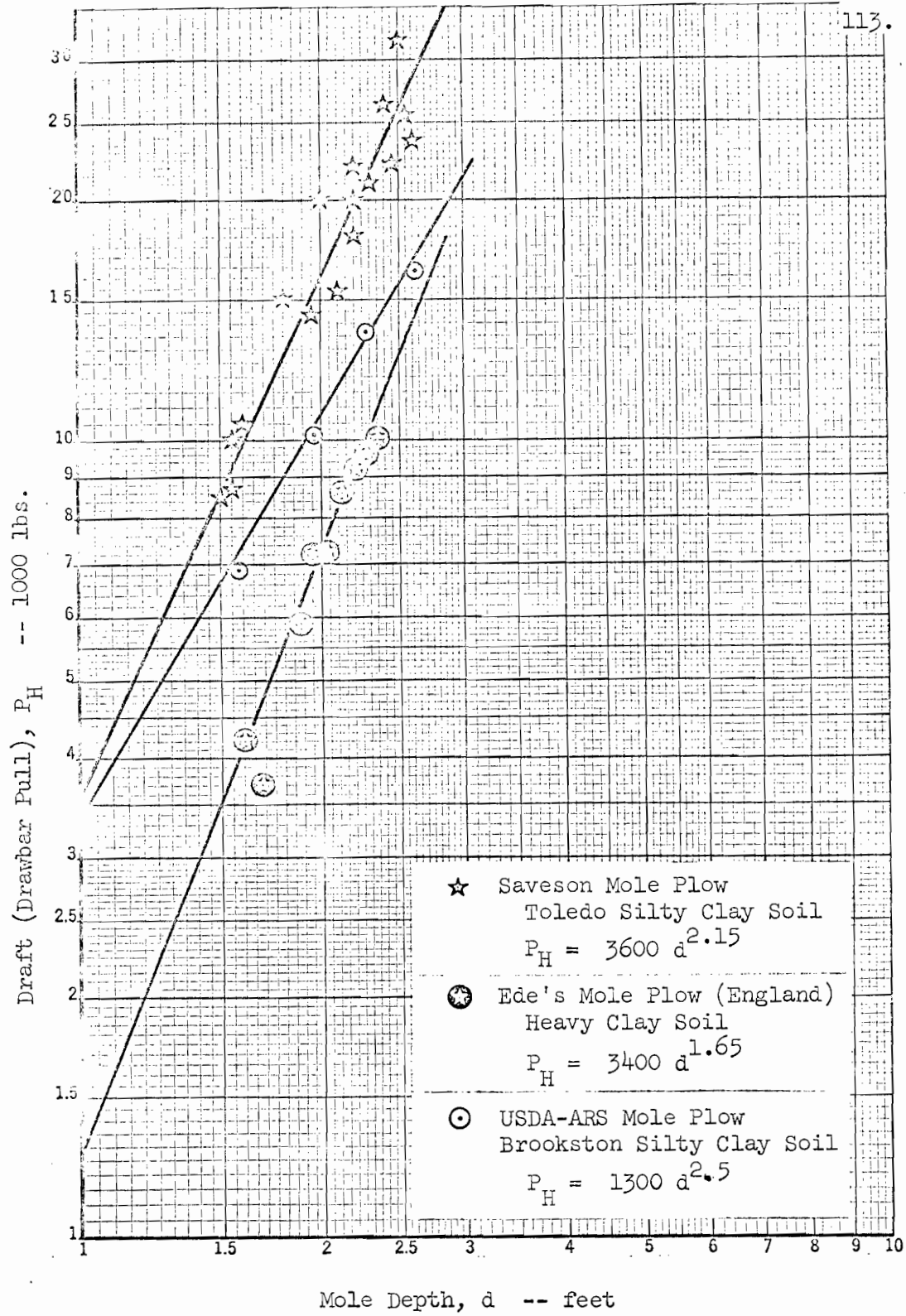


Fig. 57. Log-log plot of draft versus depth data for three mole plows.

## Ede's Long-Beam Mole Plow:

The mole plow tested by Ede (1958) was an experimental model which was equipped with a dynamometer to directly measure horizontal force (draft) on the blade. The blade size was 1 inch by 8 inches with a 3-inch diameter mole of 21 inches length. Draft versus depth data, which was plotted in Figure 57, are given in Table 6.

TABLE 6. Field Test Data of Draft versus Depth  
for a Long-Beam Mole Plow Operating in  
a Heavy Clay Soil. [Ede (1958)]<sup>1/</sup>

Moling Depth d (ft.)	Measured Draft $P_H$ (lbs. drawbar pull)
1.60	4,200
1.68	3,700
1.88	5,900
1.96	7,200
2.04	7,250
2.12	8,600
2.21	9,200
2.28	9,600
2.36	10,000

<sup>1/</sup>Data points were scaled from a graph given by Ede (1958, fig. 3).

#### Saveson Mole Plow:

The Saveson mole plow operates in a floating-beam manner, but the hitch height is constant. The depth is adjusted by hydraulically changing the relative angular positions of the hinged beam, as shown schematically in Figure 58, where the assumed line-of-draft at four beam positions is defined. The plow was field tested by Fouss (1961), and pressure in the hydraulic cylinder restraining the hinged beam was measured for several depths of operation in a Toledo silty clay soil (Table 7). The computed draft values are plotted in Figure 57.



TABLE 7. Draft versus Depth Data for Saveson Floating-  
Beam Mole-Drain Plow. <sup>1/</sup>

Mole Depth d (ft.)	Recorded Cylinder Pressure P (psi)	Press. Corr. for Static wt. at Hitch (p-100) (psi)	$\left(\frac{l_C}{l_H}\right)$	Drawbar Pull P (1000 lb.) <sup>2/</sup>	Draft Line Angle $\phi$ (deg.)	Draft Force $P_H$ (1000 lb.) <sup>3/</sup>	Vertical Draft Force $P_V$ (1000 lb.) <sup>3/</sup>
1.95	900	800	0.74	14.8	13.8	14.35	3.52
2.00	1200	1100	1.10	20.8	14.0	20.18	5.03
2.10	900	800	0.79	15.9	14.4	15.36	3.94
2.30	1100	1000	0.87	21.8	15.2	21.04	5.71
2.20	1000	900	0.83	18.7	14.8	18.06	4.77
2.20	1200	1100	0.83	22.8	14.8	22.02	5.81
2.20	1100	1000	0.83	20.8	14.8	20.09	5.30
2.20	1100	1000	0.83	20.8	14.8	20.09	5.30
2.20	1200	1100	0.83	22.8	14.8	22.02	5.81
2.40	1300	1200	0.91	27.3	15.6	26.32	7.34
2.50	1500	1400	0.95	33.1	16.0	31.81	9.10
2.45	1100	1000	0.92	23.1	15.8	22.22	6.28
2.55	1200	1100	0.97	26.7	16.2	25.6	7.42
2.60	1100	1000	0.99	24.8	16.4	23.76	6.99
1.60	800	700	0.61	10.8	12.4	10.53	2.31
1.55	700	600	0.59	8.9	12.2	8.69	1.88
1.50	700	600	0.58	8.7	12.0	8.51	1.81
1.55	800	700	0.59	10.4	12.2	10.15	2.19
1.80	1000	900	0.69	15.5	13.2	15.08	3.53

<sup>1/</sup>Source: Fouss, J. L. 1961. Field test evaluation of A  
Saveson mole plow. Unpublished research report.

<sup>2/</sup>Cross-sectional area of hydraulic cylinders is 25 in.<sup>2</sup> total;  
 $T_C = (p-100)25$  ;  $P = (l_C/l_H)T_C$  ;

<sup>3/</sup> $P_H = \cos \phi P$  ;  $P_V = \sin \phi P$  .

## USDA-ARS Mole-Drain Plow:

Field tests were conducted at The Ohio State University Farm on a Brookston silty clay soil. No attempt was made to conduct comprehensive and replicated field tests with the plow to define draft relationships for a wide range of conditions. The draft tests made, however, were confined to a small area of 40 ft. by 250 ft. in a field where soil conditions were previously determined to be reasonably uniform.

The technique used was to measure the vertical component of pull  $P_V$  at the plow hitch, and then the horizontal component (draft)  $P_H$  was computed by use of the definition for the line-of-draft (Figure 59). The vertical force  $P_V$  was measured by using pressure gauges in the input line of the hydraulic cylinders which restrain the hitch point on the tractor (Figures 45 and 46); the pressure gauge readings were recorded only for steady-state operation of the plow.

To resolve the forces at the hitch, a kinematic and force balance analysis was made of the hitch adjusting mechanism as shown in Figure 60. Force balance equations for a centered hitch position ( $h = 1.17$  ft.), and for the hitch raised and lowered 0.5 ft. are shown in Figures 61, 62, and 63, respectively. It is noted in Figure 61, for the centered hitch position, that the link AH is horizontal (coinciding with  $R_H$ ) and passes through the instant center of rotation O, thus  $P_V$  is directly proportional to the cylinder force  $F_c$ . Therefore, the centered hitch position ( $h = 1.17$  ft.) was used to determine draft verses depth; moling depth was adjusted by means of the turnbuckle screws which changed the plow beam-blade angle (Appendix A, Fig. 45). Tests conducted where the hitch was either raised or lowered 0.5 ft. provided information regarding the

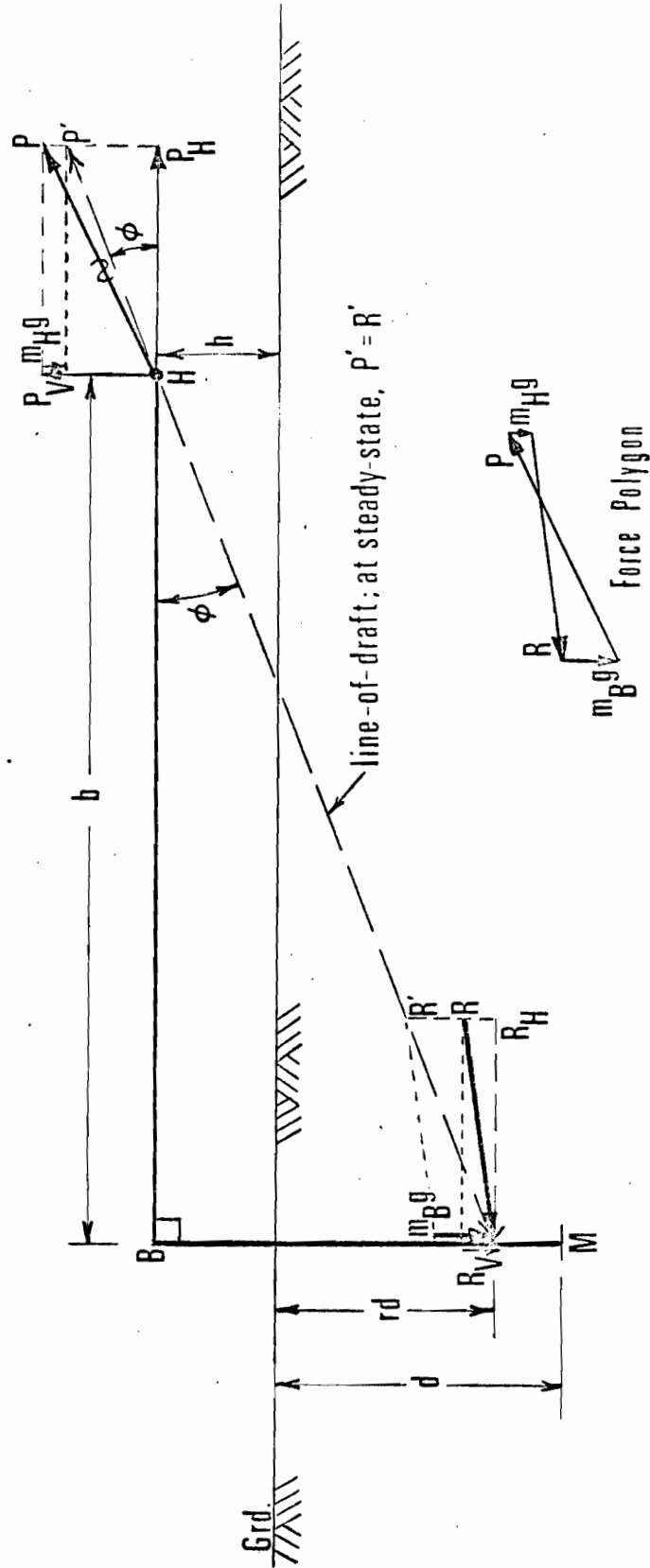


Fig. 59. Definition of the line-of-draft for a 2-lump mass model of the mole plow.

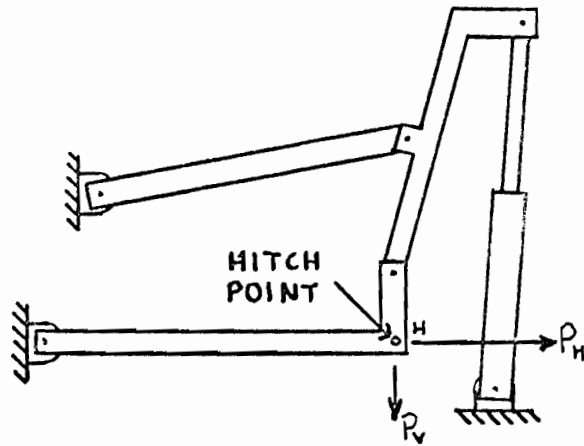


Fig. 60. Hitch control linkage for USDA-ARS Mole Plow.

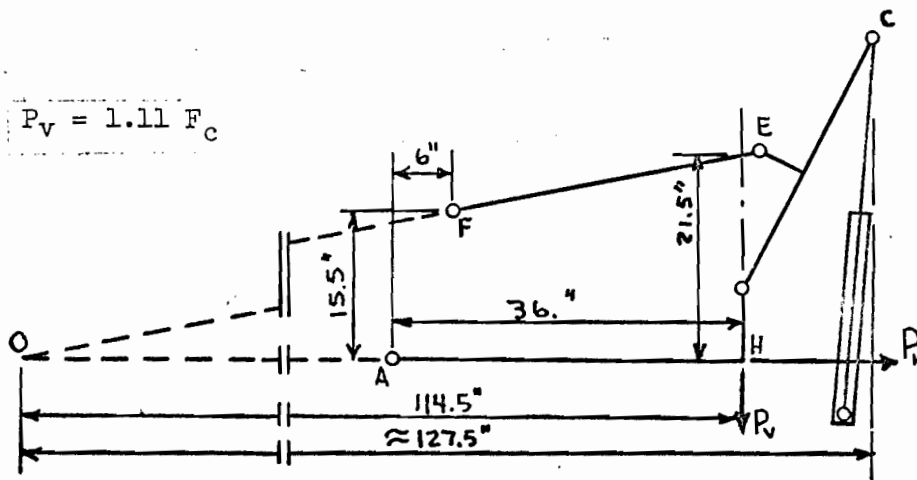


Fig. 61. Kinematic and force relationships for a centered hitch position.



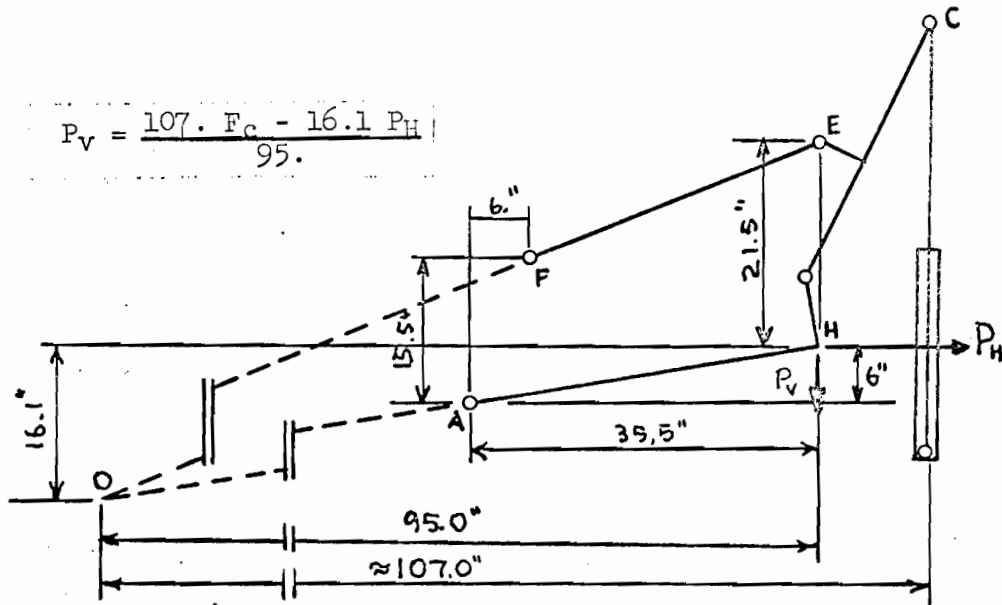


Fig. 62. Kinematic and force relationships for hitch raised 6 inches above center.

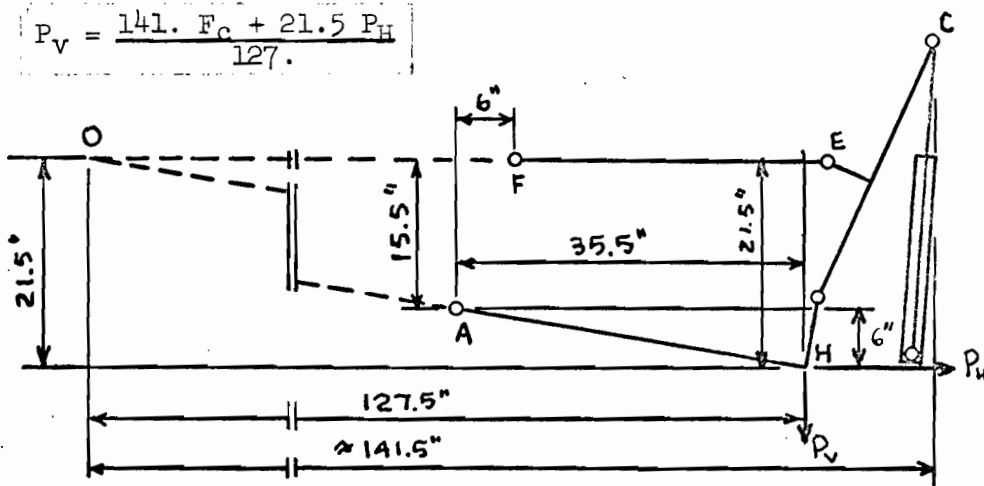


Fig. 63. Kinematic and force relationships for hitch lowered 6 inches below center.

variation in  $P_v$  for a fixed mole hitch geometry (that is, a given turnbuckle setting). Mole depth was measured with the inverted "T-probe" (Ref.: Table 3, footnote 2/, p. 98). Field data obtained for three hitch positions and several turnbuckle settings are tabulated in Table 8; the calculated cylinder force is given in the right hand column.

TABLE 8. Hydraulic Cylinder Force versus Moling Depth for USDA-ARS Mole Plow Operating In A Brookston Silty Clay Soil.

Field Test No.	Hitch Height h (ft.)	Turn-buckle Setting (no. thr.)	Measured Moling Depth d (ft.)	'Corrected' Hyd. Press. Reading <u>1/</u> (psi)	Calc. Max. Steady-State Cyl. Force $F_c$ <u>2/</u> (lb.)
1C	1.17	22	2.63	180-200	5,000
2C	1.17	23	2.27	150-160	4,000
3C	1.17	24	1.96	110-120	3,000
4C	1.17	25	1.58	≈85	2,120
5C	1.67	20	2.42	240-260	6,500
6C	1.67	21	2.21	210-225	5,630
7C	1.67	22	1.96	180-190	4,750
8C	1.67	23	1.71	140-155	3,880
9C	1.67	24	1.50	120-125	3,130
10C	0.67	24	2.73	≈50- 70	1,750
11C	0.67	26	2.17	≈ 0- 20	≈ 500
12C	0.67	27	1.83	< 0	< 0

1/ The actual initial gauge reading of  $p_o \approx 140-150$  psi included the weight of the depth adjusting linkage and a pressure gauge error; 100 psi was subtracted from the actual reading to make the lowest pressure  $p_o \approx 40$  psi corresponding to only  $m_{Hg} \approx 900$  lbs. when the tractor was not in motion and blade at a shallow depth.

2/ Total cross-sectional area of both hydraulic cylinders is 25 square inches.

Sample calculations for the steady-state vertical [ $f_V(d)$ ], and the horizontal [ $R_H(d)$ ] draft forces (using data from Test 1C, Table 8):

$$P_{VS} = 1.11 F_c = (1.11) (5,000) = 5,500 \text{ lb.}$$

Vertical:  $f_V(2.63) = P_{VS} - 4,000 = 1,500 \text{ lb.}$

Horizontal:  $R_H(2.63) = [(P_{VS} - 900)/\tan \phi] = 4,600/0.284 = 16,400 \text{ lb.}$

where,  $\tan \phi = (h_s + 0.8d_s)/b = (1.17 + 0.8(2.63))/11.5 = 0.284$

The calculated draft forces for Tests 1C, 2C, 3C, and 4C (centered hitch case) are given in Table 9.

TABLE 9. Plow Draft ( $R_H$ ) and Steady-State Vertical Soil Resistance [ $f_V(d)$ ] versus Molding Depth ( $d$ )  
Calculated From Field Test Data.

Field Test No.	Measured Molding Depth $d$ (ft.)	Calculated Draft Force $P_H = R_H$ (lbs.) <u>1/</u>	Calculated Steady-State Vertical Soil Resistance $f_V(d)$ (lbs.) <u>2/</u>
1C	2.63	16,400	1,500
2C	2.27	13,700	450
3C	1.96	10,200	- 670
4C	1.58	6,900	-1,650

1/ These data are plotted in Figure 57.

2/ Ref.: Equation [4], page 24, for  $\ddot{h} = 0$ ,  $\ddot{\theta} = 0$ .

As shown in Figure 57, the draft equation

$$P_H = 3400 d^{1.65}$$

closely approximates the data for the range  $1.5 < d < 3$  ft. Using this

draft equation, and the field data for the non-centered hitch positions (Table 8), the kinematic relationships shown in Figures 62 and 63 yield the vertical pull at the plow hitch, from which  $f_V(d)$  can be computed. A sample calculation is given below for field Test 7C, where the hitch was raised 0.5 ft. (Fig. 62) and the moling depth was 1.96 ft.:

$$R_H(1.96) = 3400 (1.96)^{1.65} = 10,300 \text{ lb.}$$

$$P_{VS} = \frac{107 (4,750) - 16.1 (10,300)}{95} = 3,600 \text{ lb.}$$

$$f_V(1.96) = 3,600 - 4,000 = -400 \text{ lb.}$$

The calculated values of  $f_V(d)$  for the field Tests 7C, 8C, 9C, and 10C are given in Table 10. Figure 64 presents graphically the theoretical relationship between  $f_V(d)$  and  $d$  (that is, equation [8], page 26, for  $\dot{d} = 0$ ), and the calculated values of  $f_V(d)$  given in Tables 9 and 10. It can be seen that the values calculated from the field data, and using the estimated beam length of 11.5 ft., agree reasonable well with the theoretical curves for different plow configurations.

TABLE 10. Steady-State Vertical Component of Soil Resistance [ $f_V(d)$ ]  
Calculated From Field Test Data and Emperical Draft Equation.

Field Test No.	Measured Moling Depth $d$ (ft.)	Plow Geometry Factor $n$ (ft.) <sup>1/</sup>	Calculated s.s. Vert. Force $f_V(d)$ (lb.)	Plotting Designation for Data [Fig. 64]
7C	1.96	3.25	- 400	◻
8C	1.71	3.01	-1,030	△
9C	1.50	2.82	-1,600	⊙
10C	2.73	2.82	940	⊙

<sup>1/</sup>Ref.: Appendix B, Table 3, and Figures 49 and 50.

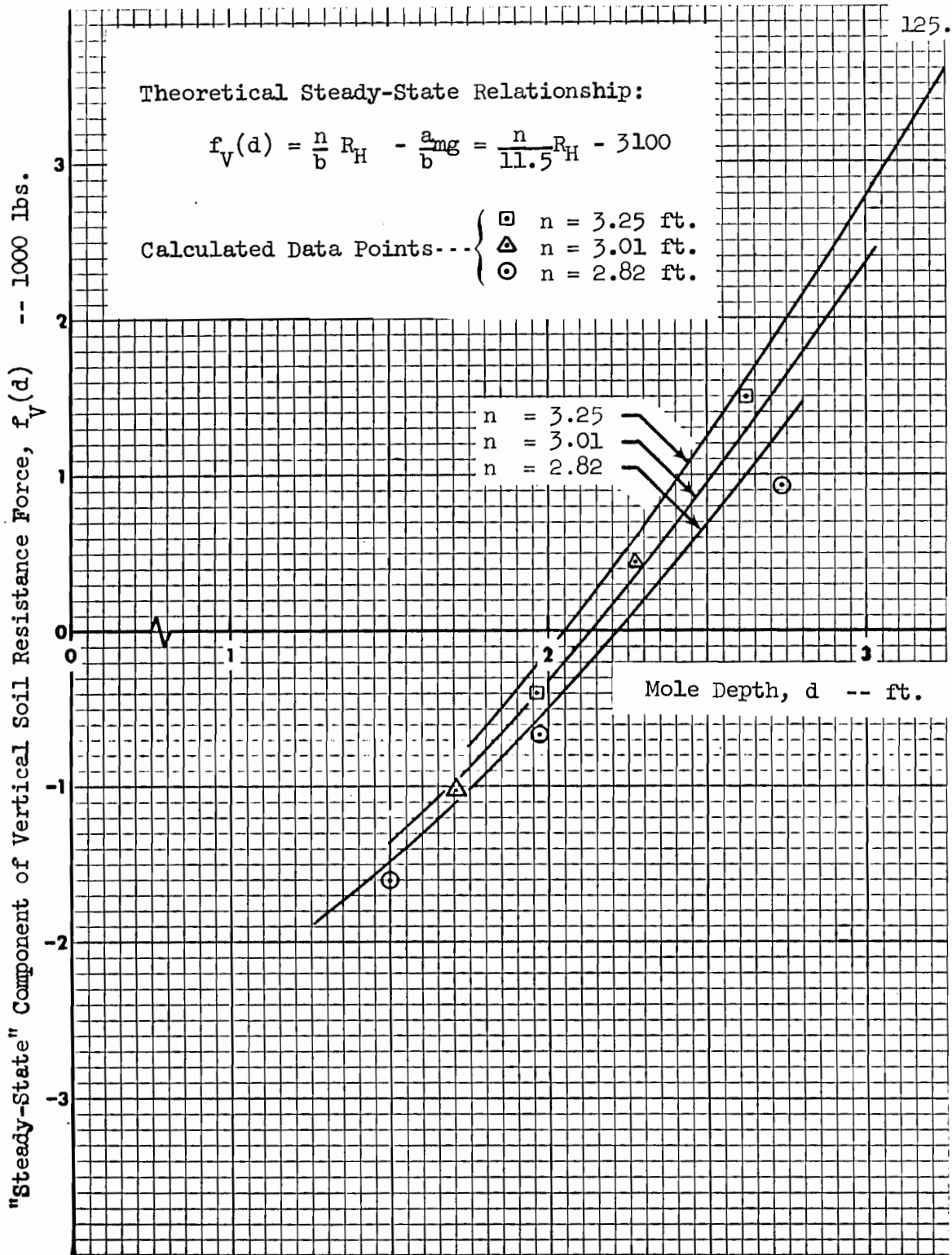


Fig. 64. Comparison of theoretical and measured steady-state vertical soil resistance force versus moling depth.

Since the developed draft equation for the USDA-ARS mole plow (Fig. 57) was only considered applicable for the range  $1 \leq d \leq 3$  ft., the draft force for  $d < 1$  ft. was estimated as follows: It was assumed that the minimum draft (at  $d = 0$ ) is equal to the sliding friction force between the mole torpedo (M) and the ground surface. At  $d = 0$ , the rear reaction force at M is  $m_B g \approx 3100$  lb., and assuming a coefficient of sliding friction<sup>1/</sup>,  $\mu = 0.5$ , the sliding friction force can be estimated as:

$$F = \mu m_B g = 0.5 (3100) = 1500 \text{ lb.}, \text{ for } d = 0 .$$

The graphical draft relationship between  $0 < d < 1$  ft. was estimated by "eye". The draft versus depth relationship programmed on a variable function generator in the computer simulation (Chapters II and III) is shown in Figure 65.

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<sup>1/</sup>Gill and Vanden Berg (1967, p. 52, fig. 38).

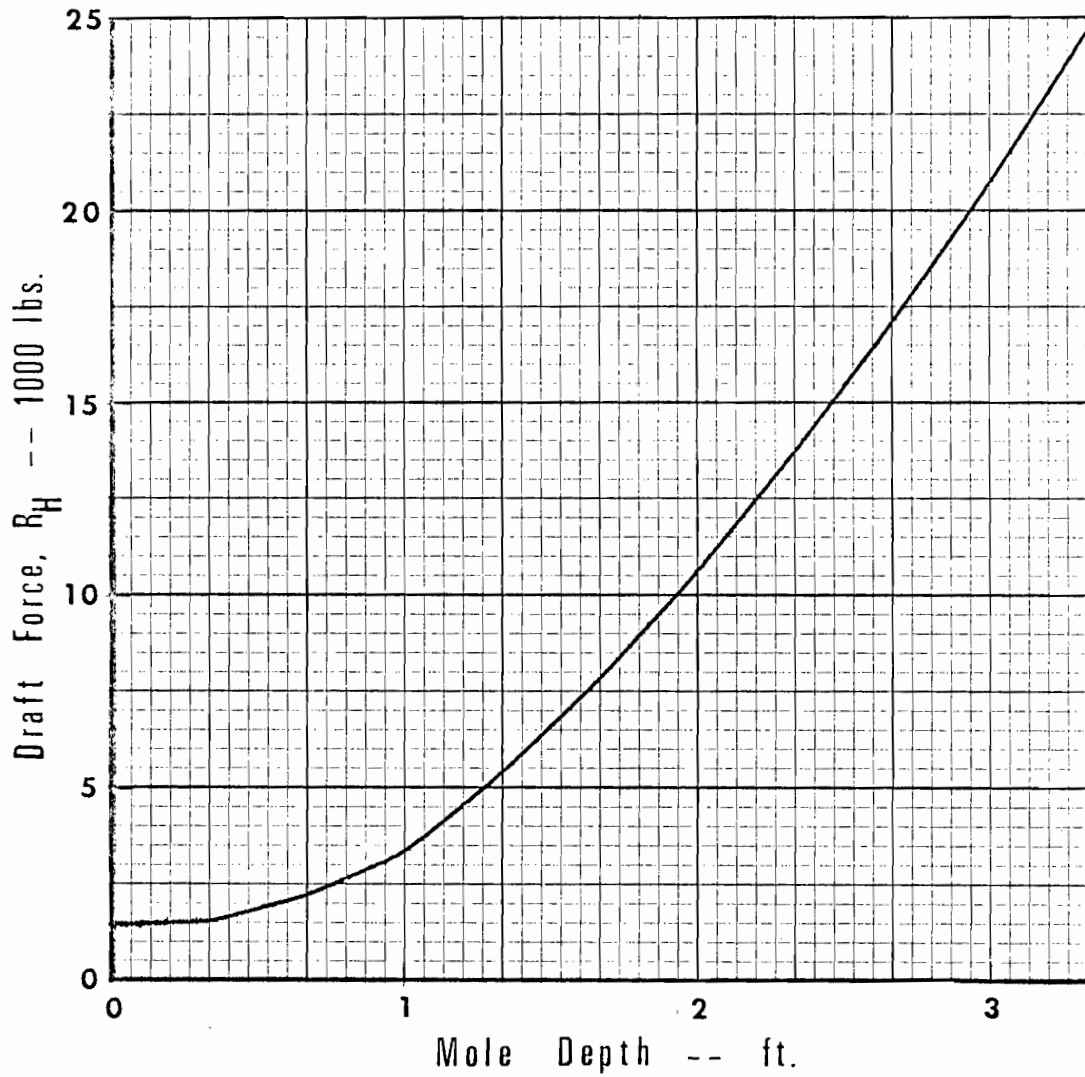


Fig. 65. Draft versus depth relationship programmed for computer simulation.

APPENDIX D



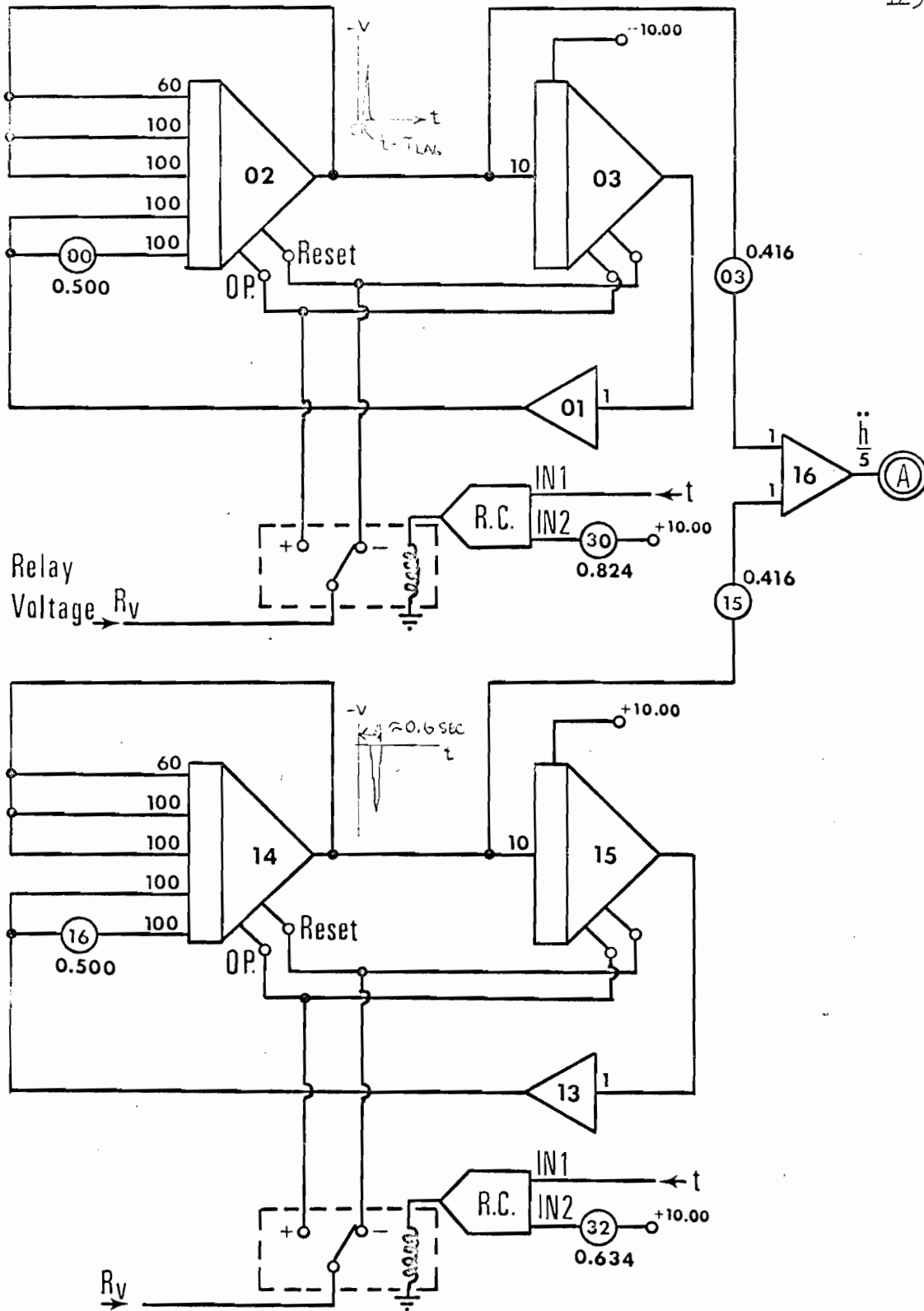


Fig. 66 . Analog computer circuit to simulate acceleration and deceleration pulses of the mole plow hitch point caused by motion of the hydraulic depth adjusting cylinders.

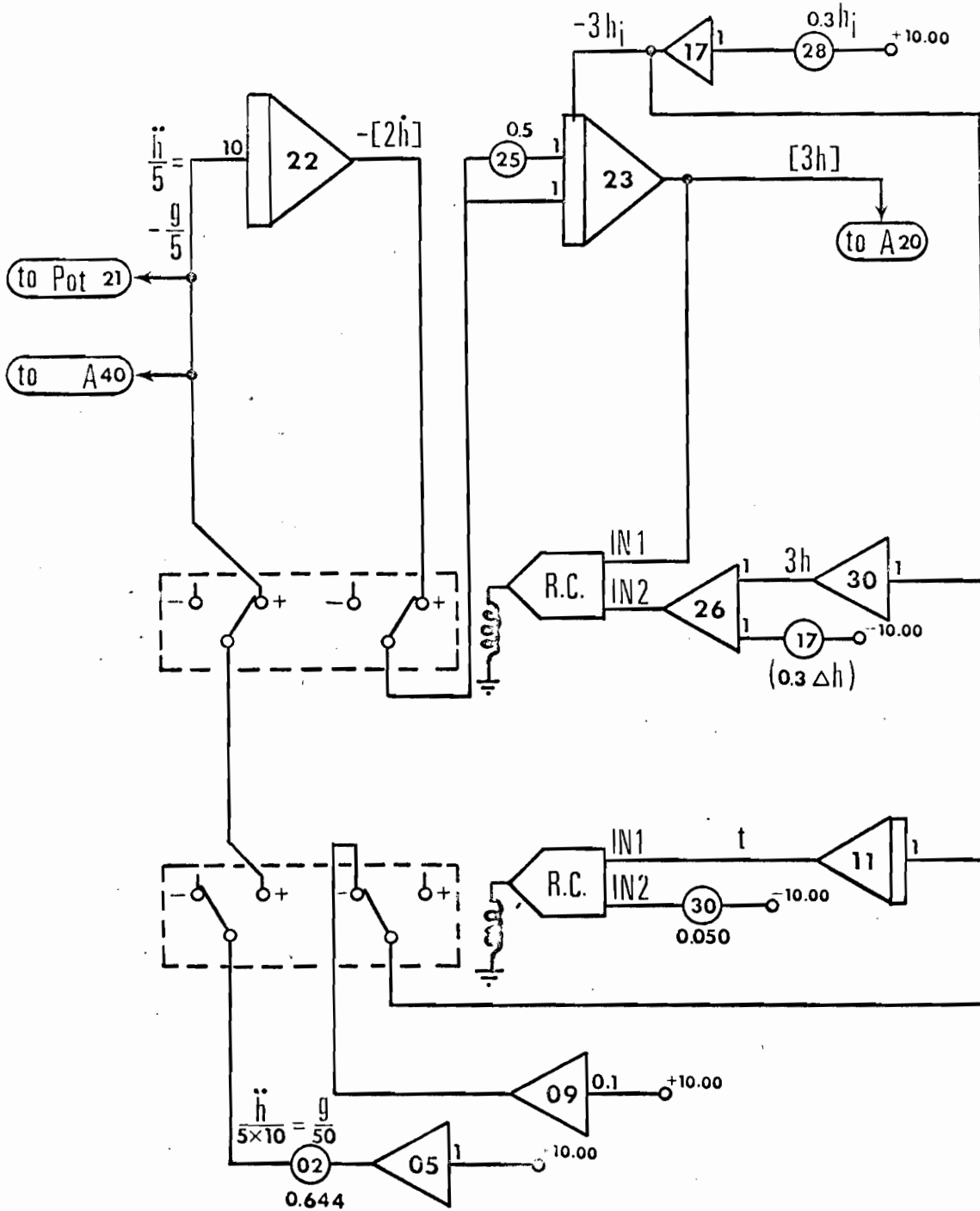


Fig. 67. Analog computer circuit to simulate gravitational "free fall" of the mole plow hitch point.

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